Editorial

M. Asoka T. de Silva

Mathematics as a Central Science

Mathematics while being central to other scientific disciplines can also be considered to be one of the most ancient of abstract sciences comprising numbers and quantities, which contributed to the cultural traits of evolving civilizations.

The significance of mathematics is seen in the number of anthropocentric investigations by pre-medieval, medieval and contemporary scholars. These philosophers and writers were no doubt in search of the origins of the constituent branches and sub-disciplines that now constitute the central discipline called mathematics.

It is known that throughout the first millennium and in a greater part of the second millennium, Greeks, Persians, Arabs, Indians and Chinese played a decisive role in the cross-transmission of elements of thought, philosophy, culture, theosophy and learning, among which were notations, symbols and concepts of arithmetic.

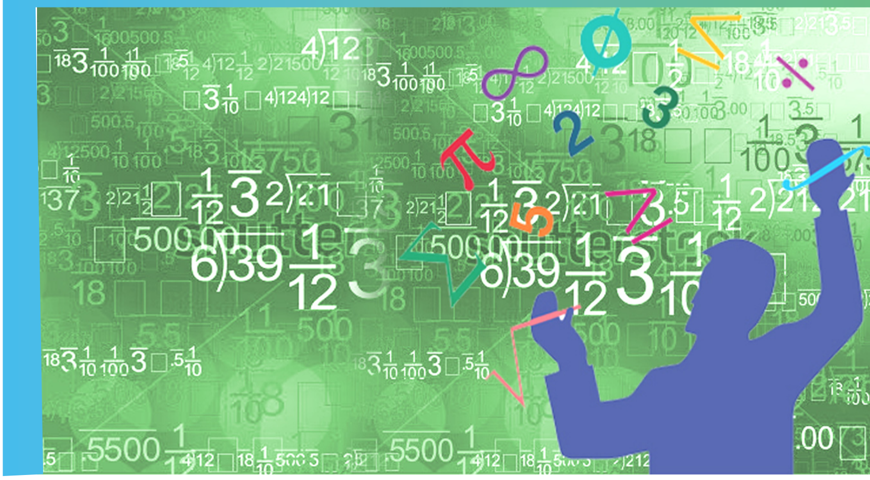
Susantha Goonetileke in his scholarly analysis of what has been described as 'Civilizational Knowledge' published under the title *'Towards a Global Science'* (1998), comments that Arabs in ancient times referred to mathematics as *Hindsat*, implying it to be the "Indian Art", illustrating their own perceptions of the prescriptive thoughts on mathematics. The mathematical knowledge borrowed from Indian sources included the Indian method of mathematical notations, concepts of zero, the decimal system, algebra and trigonometry.

Subsequently the acquisition of this knowledge by Europeans, believed to be through Greek sources about the beginning of the Renaissance (ca. 14th Century), led to the development of the new arts of navigation, astronomy and weaponry of warfare. According to Goonetileke, this new knowledge included significant borrowings from the East, especially of mathematics. Consequently the South Asian systems of arithmetic, algebra and trigonometry were developed rapidly and utilized by the Europeans in their so-called voyages of discovery.

Significantly, it is said that Europe had taken a long time to accept the idea of a zero, this being considered a non existent intangible entity or number.

The current edition of *Vidurava* which is devoted to mathematics, has articles that illustrate the versatility of the subject through popular applications of this discipline in the day to day life of people. Its usefulness in facilitating formulation of the most logical and prudent answers and decisions in relation to common concerns that affect livelihoods of the common man has been the focused intention of these authors.

Mathematics: Resolution of day today Problems



Mr B. D. Chitthananda Biyanwila

Civilization and culture are built upon the cumulative results of the attempts that humans have made to solve the problems faced from time immemorial. In trying to solve the problems that humans faced either as individuals or as groups using their knowledge, intelligence, experience, methods and equipments, they have been able to introduce new methodologies, experience, success, identify the background theories and conventions, and also develop new fields of knowledge. Due to this inherent skill of humans, today we have inherited civilizations rich in many areas such as religion, Science, Mathematics and Arts.

Problems were solved according to their nature, and to further develop the relevant areas more and more similar problems were solved. Solution of new problems of similar nature helped in this development. In this manner the problems of communication through language, receiving blessings, protection and salvation through religion, understanding the individual causes through science and problems of quantity, shape and space through mathematics have been resolved. But it is necessary to investigate the extent and the manner in which we have referred to and used the wealth of knowledge in order to solve the problem we are now faced. It is more important to investigate whether the field of mathematics which is an abstract subject is being used adequately and fruitfully to solve the day to day problems.

Mathematical applications providing solutions

If you give a copy of this Vidurava magazine to a friend in order to read this article the problem arises as to how he is going to locate the article in the magazine. The 'page number' given through the numbering notation will help him to determine whether he should turn the pages forwards or backwards from the page which is already opened. The mathematical understanding of the sequence of numbers will help him. Even though it is planned to print 5000 copies of the book, and if only 3000 copies have been printed so far, then we know that 2000 more copies have to be printed. It is because numbers are used to indicate the quantity and mathematical operations are carried out on these numbers as required, and a simple method has been developed to do this, that we are trained to solve the problem without hesitation .

But there are instances when mathematics alone cannot provide meaningful solutions in isolation. When finding out whether 5kg of sugar has more mass than 10g of sugar, the concepts pertaining to dimensions and units in physics, and the mathematical concepts associated with numbers have to be used in combination. Otherwise comparing the numerical values will not yield the correct solution. This very simple example explains that if the correct solution has to be obtained it is necessary to use consistently and correctly the mathematical concepts along with the other scientific and social concepts.

Therefore in the simplest situation of using numbers to indicate quantity and also using numerals as labels in order to arrive at the correct solution it is necessary to be aware of the associated norms, standards and interpretations. When investigating the fever of a child, if a reading of 101.5°F was obtained, to understand that it is a high value causing problems, it is necessary to have known that the normal healthy temperature should be 98.4°F. When examining the health of a person the medical reports providing the values for a particular substance present, the rates,

concentrations etc. are analysed and a decision is talees as to whether these values fall within the range of standards and indicative of sound health depending on his age and weight. Here it is clear that to minimize the health problems it is immensely useful if the simple knowledge of mathematics can be used to analyse them.

Solutions with mathematical explanations

In order to free one self of obesity which is detrimental to health it is seen that people show an interest to control their intake of food and engage in exercise. If one is to calculate the amount of calories contained in the food consumed and the calories that should be expended by exercise, then it should be possible to control it without resorting to costly medical advice.

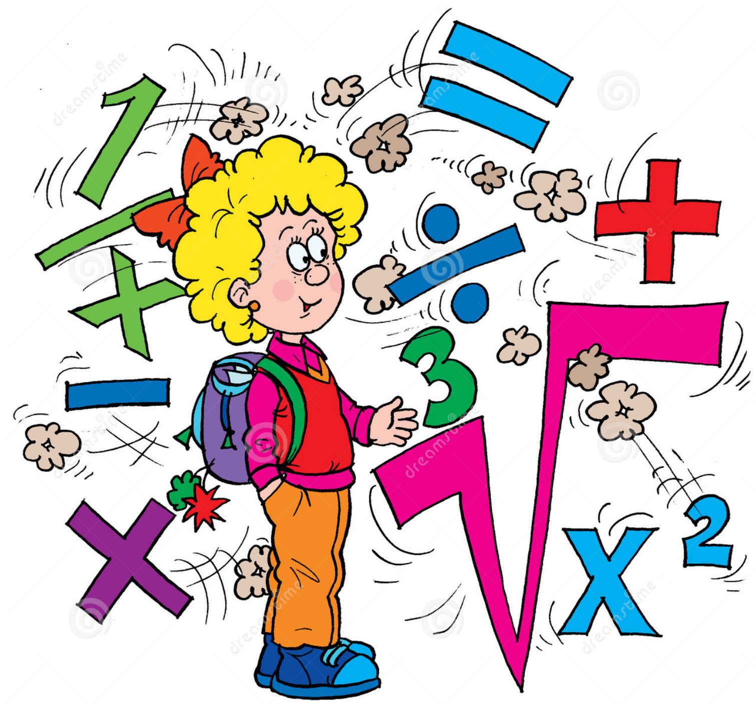
If you know the relationship between the octane number and the composition of fuel, then to decide whether you should use octane 92 or 95 for your vehicle would be easier. This knowledge of notations relating to proportions will be useful to you even when making gold jewellery according to you requirements, and it would be beneficial if you have some understanding that gold is categorized on 24, 22 or 18 carat gold depending on the proportions of the metals gold, silver and copper. You will be able to advise the gold smith to make the gold jewellery to suit the cost and requirement if you possess the scientific knowledge of physics and chemistry. For example by increasing the proportion of copper the malleability can be increased, and the brightness can be increased by increasing the proportion of silver a long with it, if you possess the mathematical knowledge of proportions.

There exists useful meaning even in mathematical applications with geometrical basis. Imagine that a vesak lantern designed as *'Atapattama'* has been made during the vesak season. Can you say what the eight frames are in this particular lantern? If the *'Atapattama'* is made using 6 frames which are squares, then it will not be strong and firm. The *'Atapattama'* has to be made by making eight triangular frames first and then joining these up as required. It is possible to make the triangular frames to be steadier and firmer than the square frame. Thus through correct understanding the messages embodied even in mathematical terms, it is possible to find a solution to a *'Atapattama'* that is shaky.

Abstract mathematics and the day to day world

In the process of the wide development in the field of mathematics from simple mathematics and Euclidean geometry, which are the corner stones of modern advanced mathematics comprising a variety of forms, mathematicians have made many discoveries and found solutions to problems in ordinary life. These discoveries and solutions are of a higher level than necessary for provision of solutions to problems, and can be thought of as belonging to a wealth of abstract knowledge. It is not unreasonable to consider the problems of day to day life and mathematical problems as different. The day to day world is practically experienced and perceived by the senses, and the mathematical world is built on the basis of abstract concepts that are clearly conceptually different from each other. This is because we understand the two worlds is a different manner, and what is important to us of the two worlds is different. However, the finding of solutions to day to day problems and to mathematical problems are processes that cannot be matched and that itself may evade the path to the solution.

In the day to day world null is considered as something that is non existent with no meaning. In this mathematical world $\sqrt{-1}$ is denoted as i and is considered real. It is identified as an imaginary number, and even though imaginary it is an important number in the mathematical world. Similarly, in the mathematical world there are the geometrical entities such as the point and the straight line. Even though the point and the straight line have been deviced with their characteristic features, they exist in the mathematical world and not in the day to day world. The point with its characteristic of having no width or breadth do not belong to the world of general sensory perception.



The existence of concepts which are not perceived by senses, and their very abstract nature, qualifies mathematics to be considered as a universal language. Radio astronomers believe that it may be possible to use the mathematical language to communicate even with living beings of outer space who may possess a different organ system because mathematics is based on natural laws which are associated with quantity, shape and space - the characteristics that intelligent beings and the universe may be able to grasp and is composed of abstract concepts. Radio astronomers of the Green Bank National Radio Observatory in West Virginia have aimed two giant disc antennas at the two stars Tau Ceti and Epsilon Eridant for the past several years, and are waiting to receive a message of electromagnetic waves based on this mathematical language.

Solution of problems using mathematical language skills

In order to successfully use this mathematical language, firstly one has to know its vocabulary and the grammar. The grammar is none other than mathematical logic and the system of mathematical laws. The vocabulary is the system of mathematical symbols. To denote the numbers digits are used, for the unknown numbers letters named algebraic terms are used; for mathematical operations of addition, subtraction , multiplication and division symbols such as **+, -, ×,** ÷ are used; to denote equality and unequality symbols such as =, ≠, <, > are used; to denote intersection and union symbols **∈** And **∪** are used ; to denote square root and infinity the symbols $\sqrt{\ }$, ∞ equations and so on are used. This vocabulary of mathematics is composed of a limited number of mathematical symbols, some of which are as follows. Your ability to solve the problems associated with day to day life will be enhanced to the same extent as the skills you possess to manipulate the mathematical language based on this mathematical grammar and vocabulary. We would not have claimed ownership to this civilization if not for the development of intelligent technologies and physical laws which are by - products of mathematical research. Therefore it is justifiable that the current trend of mathematics which has developed along several branches should not be distanced from the problems of ordinary life considering it to be an abstract subject perceived only by mathematicians. The wiser thing to do is to focus attention on the possibilities of finding solutions directly or indirectly by using mathematics, as indicated below. Even if the mathematical skills relevant to the subject of mathematics in the secondary school only have been acquired, it will be sufficient to find solutions for day to day problems that discipline of mathematics can solve.

Application of various mathematical solutions

The architect who designs the plan of your house which has according to your requirements the kitchen, bedrooms, living rooms, bathrooms, verandahs etc. solves this day to day professional problems using arithmetic and geometry. Not only him but also the masons and the carpenters who get involved in constructing the house according to the plan, regularly use the measurements of plane geometry introduced by the ancient Egyptian mathematicians. Also your discussions with the house builder would be more meaningful and successful to the extent that you get informed about the relevant mathematical applications according to the time, and may be your ability to spend to build a house according to your requirements

When dealing with the mason or the carpenter and even when supplying the raw materials for building it will be possible to save time and money only if you are not hesitant to use at least to a certain extent the mathematical knowledge regarding estimates of the relevant resources.

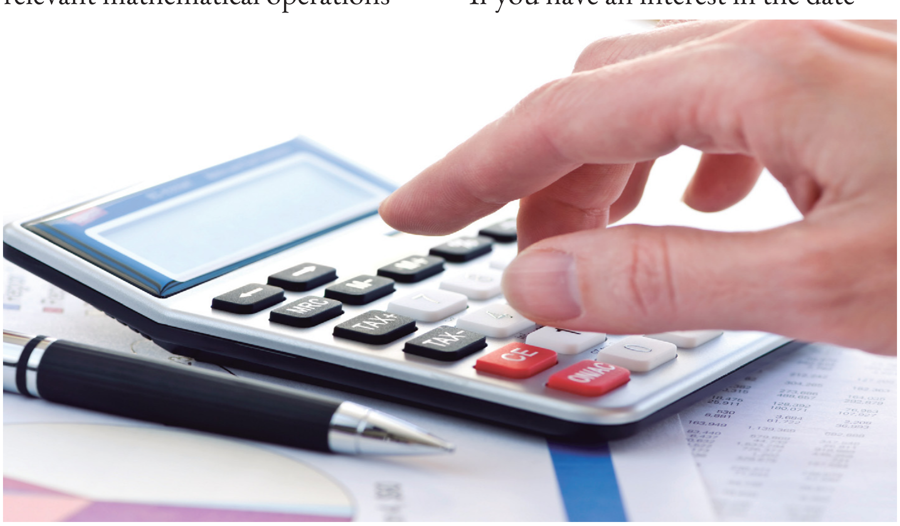
In planning massive projects of the state, which affect our life style, the planners and the officers who calculate and determine physical, financial and human resources will have to obtain their solutions through many mathematical operations. It would be possible to enjoy the social benefits accruing from these massive projects more meaningfully according to the skills acquired through the practice and experience of carrying out the relevant mathematical operations in estimating the cost and benefits. The logical selection of the super highway, and the comparison of factors such as fuel and fees, is a search for a solution to a mathematical problem. A scientific civil society which offers proposals for the development of a country, will succeed only if there are citizens experienced in mathematically analyzing, presenting the facts in a logical manner, and are not hesitant to make the required calculations.

The laboratory technician who analyses the urine sample given by you for a medical examination examines the sample and calculates the relevant values and writes them down by carrying out a large number of varied types of calculations during his daily professional life. For this purpose he may be aided by tables, graphs, calculators and computer programmes. If he is skilled in mathematics and in the use of these aids, then his service will be more accurate and efficient. However since there is a probability for reports to be inaccurate due to various reasons, it is your responsibility to examine the accuracy to the best of your ability.

If you have an interest in the date contained in your medical report and the range of values expected for a healthy person you will be motivated to make a comparative study of the value variations from the previous reports and understand them. Then you will also be able to compare the treatments you receive with the reports. The person who possesses the required mathematical knowledge will naturaly be led to do so. If it become necessary to get medical advice within a short time, after showing the current medical reports, then it would be very convenient and helpful to the doctor, especially, if you are able to present how the values in the past reports have varied with time in a summary form or using graphs . Let us consider the medical test determination where the estimated glomerules filtration rates (e-GFR) are determined in order to find out about kidney function. There are many methods of obtaining the e-GFR value that are followed in Sri Lankan laboratories. It is calculated using an equation consisting of four variables as given below.

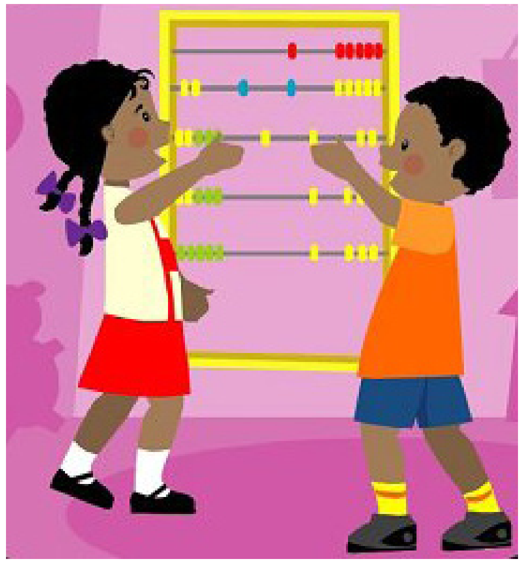
e GFR = 186 × (serum creatinine) -1.154 × age -0.203 (if ethnically black 1.21) (if female × 0.742)

It may be difficult to understand the mathematics of this, calculation but it is clear that the relevant value is dependent on four variables, the serum creatinine value, age, ethnic type and the gender of the patient. Even a person with a G.C.E. (O/L) knowledge of mathematics with a knowledge of negative indices will be able to logically understand that with the increases in the serum creatinine value and age, the e-GFR value will increase. By studying the reports it is possible to of find out that for a chronic kidney patient the



result obtained by application of the equation is valid and a e-GFR value of more than 60ml for a healthy person, the e-GFR value obtained by applying the equation may be a little less. However such an understanding is possible with practice and experience of analysing this type of information associated with mathematics, and using the equations with confidence and without fear. This will give you the ability not only to understand the information in the medical reports but also the ability to assess the validity of the medical reports. Due to the reason of not paying sufficient attention to situations such as this where application of mathematics should be close to one's day to day life a communication barrier between the doctor and the patient exists and it is necessary to remove this barrier.

It is the simple arithmatic designed by the ancient Phoenicians and the Indus valley civilization that helps to formulate the country's annual budget report which determines our daily life style including the daily meal and also helps us to check the accuracy of the bills we have paid in a shopping complex for purchasing the day to day requirements although there are calculators which make this arithmetic to be done easily, in practical situations you will not be able to do it unless you have the practice and experience of doing the arithmetic yourself. There are the entrepreneurs and consumers who use mathematics to arrive at correct and profitable decisions to make investments. People who are accustomed to using mathematical knowledge in their day to day life are capable of making



the correct assessments regarding the payment they should make for goods and services. It is because of the reluctance to use mathematical methods such as simple arithmetic, probability and statistics that people get confused. This is due to their inability to arrive at correct decisions regarding the acceptance or rejection of various proposals, based on analysis and calculation of true benefits arising from various forms of discounts, insurance schemes and lease systems.

Mathematical process of arriving at solutions

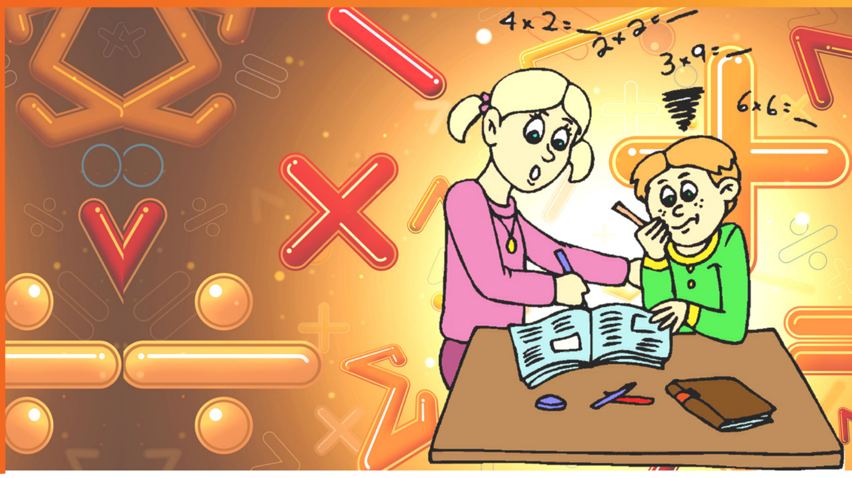
Even a person who knows nothing about cookery can say whether a curry has too much or too little salt. Likewise there are situations when even if a person cannot make the entire calculation, if he understands how the calculation is done he might be able to infer the expected answer. However even that will not be possible until he exammes logically the way the calculation is done.

However if you are willing and prepared to try to solve the problem mathematically, the first step is to translate after analyzing the problem given in the normal language into the mathematical language. On analysis of the problem using your knowledge of mathematics you will be able to guess the likely theories, and laws that need to be used. Then you will be able to arrive at a solution by carrying out the required mathematical operations using the relevant mathematical symbols and formulae. If the solution is successful then you can accept it. If not you must attempt to do it applying some other laws. Sometimes the existing mathematical theories may not be adequate to solve the problem. It is in situation like this that the mathematicians come up with new theories to solve problem.

It is due to the development of science and technology in the fields of Physics, Chemistry and Biology resulting as by – products of mathematical research that the present human civilization and culture in which you and I exist have come into being. Even though we are still unaware of the nature of mathematical research that was carried out, the technology for the construction of Sri Lanka's reservoirs (tanks) and Dagabas, textile technology of India and the printing technology of China serve as historical evidence for the application of mathematical methods in solving the day to day requirements of people. Therefore you should not hesitate or delay in making mathematics your assistant to solve the day to day problems.

Mr B. D. Chitthananda Biyanwila
Director – Mathematics
Ministry of Education

Mathematics in day to day life



Anuradha Mahasinghe

It is seven ‘o clock on the dot. After a short while, I would fasten the door and then I am supposed to be engulfed by the tensive morning road traffic. Yet I have forgotten something. Still I am on bathroom slippers, not my office shoes. I stare at the doorway. Shoes are right here. Still I need to look for a pair of socks in the clothes cabinet. It is not an easy task, as the socks of different types are mingled with all types of garments inside. This hardness is often expected in this sort of untidy environment.

After a few unsuccessful trials, I become tired and then get mad. The worst possibility is ending up with no single pair of socks. No other incident is better than this type of simple misfortune, if you want to lose your temper over something. I get mad over my failure; because I believe that finding a pair of socks is quite natural, though it did not happen so. In other words, why did I end up with the worst, or the least probable; while I could have ended up with something better, and seemingly more probable? If you allow cultural believes to mingle up with thoughts, I would call it fate.

Occurrence of the less probable always reminds us of fate. When an airplane crashes in the air and disappears into the sea, it is explainable in terms of the fate of the pilots, the cabin crew and the passengers, yet we would consider it as a natural phenomenon, if it landed at the destination safely. Though we suppose the probability of being disappeared is very low for an airplane, because not a single airplane has disappeared in history. That implies, irrelevance of its probability to become true, an incident with a non-zero probability could somehow become true one day. After being true, the fact that it had a low probability does not make sense to us anymore.

On the other hand, is it fair at all to conclude that something has zero probability merely by past information? Can you say the sun will rise tomorrow, just because every day you observed that it did? According to a Russian fairy tale, once upon a time the bear swallowed the sun. The other animals were looking for the bear in each and every corner of the blacked-out world! Thus, the non-physical entity of sun could easily be endangered even by a storyteller.

Even the physical is neither absolute nor independent of conditions. It is a physical body composed of matter and energy, and always subject to change. Therefore, why should we expect the sun to rise tomorrow without doubt?

In a practical perspective, this questions the validity of the applied form of probability. In this sense, if something can happen at all, the possibility is primary; while the probability becomes secondary. (This is somewhat analogous to existence and uniqueness theorems in mathematics.) So, the worst possible outcome could become true someday. One of these days, if followed by an unusual energy transition, the sun may not rise. The most sophisticated aircraft will undergo technical faults in the air and may disappear. Finally, I would end up with singles, no matching pair of socks.

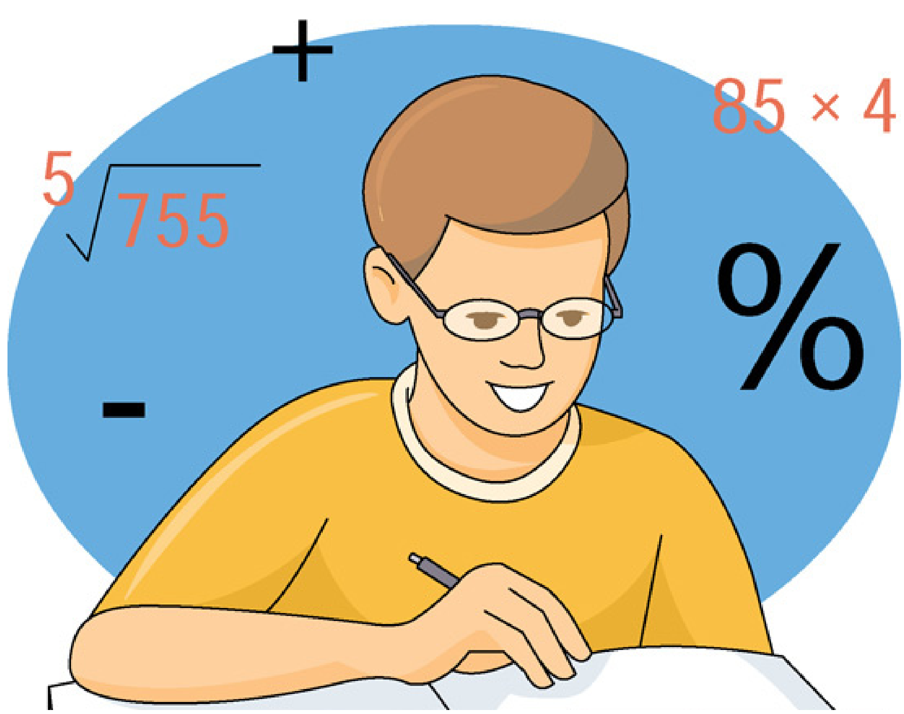
Murphy’s Law, the epigram with the meaning that if anything could go wrong, then it will enter into the diction of the English language, neither will it be for mathematics nor for the probability theory. Absence of tomorrow’s sun, disappearance of a modern aircraft and ending up with no pair of socks are a few applications of this so-called law. This law is attributed to Edward Murphy, an American aerospace engineer who passed away about twenty five years ago. It is said that Murphy uttered the following words, referring to a reckless worker at his laboratory: “If there is some way to do it wrong, he will”. However, the most popular version of Murphy’s Law is the saying “if something could go wrong, then it will”. This addresses the primary-secondary issue of possibility and probability of an incident.

Sufficient evidences prove that the famous mathematician De Morgan (De Morgan’s rules in set theory are named after him.) was aware of this phenomenon, before Murphy or his remarkable employee. De Morgan had cited, “if tested sufficiently, anything would happen”. If we observe for millions of years, then we would find a day the sun does not rise. If we continuously let more and more airplanes fly, one would disappear. If you are going for work in shoes and socks for many years, one day you would miss a pair of socks.

Robert Matthews, a prolific science writer, elaborated Murphy’s Law by using this socks dilemma. Suppose I got ten pairs of socks, and six socks have gone missing. Though I am never pleased to lose my socks, there’s a way I would like most to let that happen: the missing six make three complete pairs. Then I would have seven pairs left with me. What I dislike most is, the missing six belonging to six different pairs. Using the basic counting principles and elementary probability theory, the probabilities of these best and worst outcomes could be calculated. Amazingly, the probability that the missing socks are from three different types is about 0.003, while they have a probability of 0.347 to belong to six different types. So, the worst is about hundred times more probable than the best, supporting the pessimism of Murphy.

Two minutes late than usual. I must somehow walk to the bus stop as soon as possible. So, let’s not stick to the rule that you always need to put on the same type of socks. Two socks from different colors; it is not so funny after all. Just as I fasten the door, the mother cat appeared before me. She’s a wanderer, and has a litter of three nice kittens. She would like to have something for breakfast. I open the door again, and brought some food for her. Three minutes late now. I rush along the narrow road, listening to the thanksgiving of the cat family. The old lady at the fish-stall once said to me that a cat takes her litter to seven different places. If she is true, I feel mine is the seventh. One day I saw her taking her kids from one place to another. Even after taking the last, she went back and looked for some other kid. Some people say they cannot count. Maybe that is why she did so. (How a cat counts seven is a different problem.)

Though it is not clear if man is the only living being who is capable of counting, it is undoubtedly admitted that counting is the foundation of all creations of mankind in all civilizations. Counting actually comes before literacy in any civilization. You do not need to go to the valley of Nile to find a proof. The fish-stall located about fifty meters would suffice. The fishmonger, the living proof, is an old lady. Now I’m actually very close to her place and hear her voice clearly. Though she is illiterate, nobody could ever cheat her by means of change, coins or wherever basic mathematical operations that would apply in fish-selling. As literacy is not simply that a person knows the alphabet, the mathematical literacy is not simply the capability of adding, subtracting, multiplying or dividing. (Professor John Allen Paulos, a prominent writer on non-textbook mathematics refers to the illiteracy of very basic mathematics as Innumeracy.) Maybe



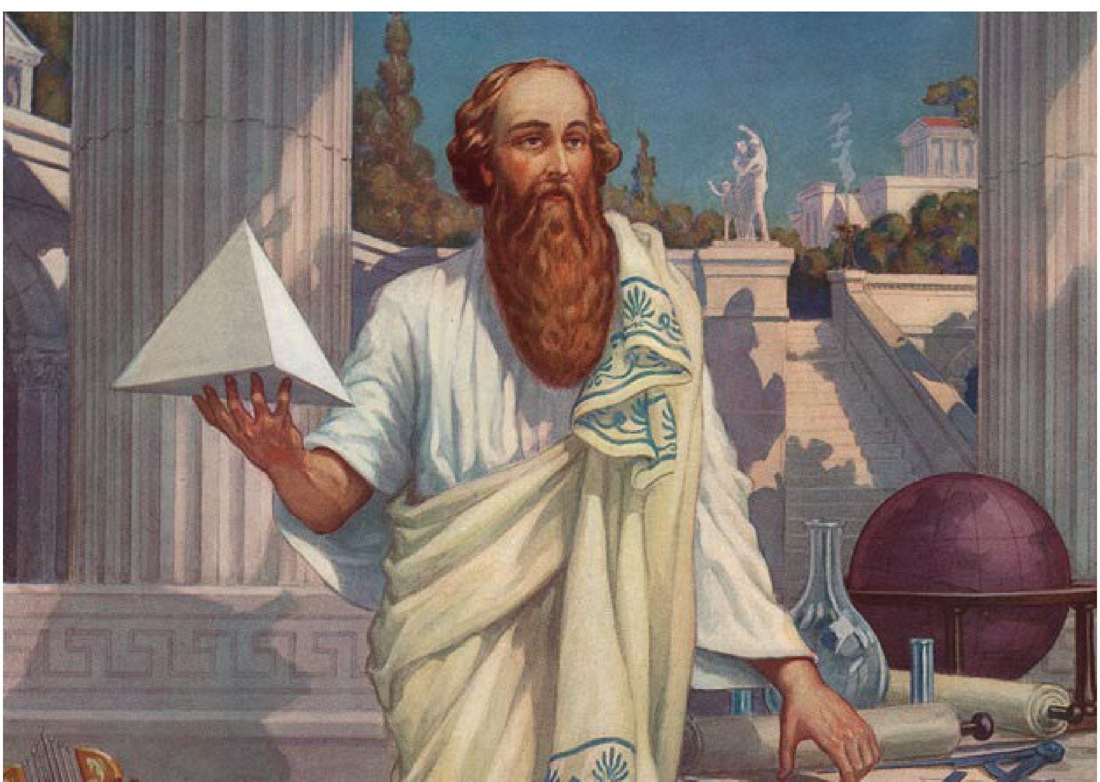
illiterate, but she is not innumerate, in this Professor's jargon. Mathematical rules are eagerly applied in her fish-stall daily. These include Proportional laws, decimals, interest rates etcetera. If you buy tuna and sprat together, you would get a discount. Two people buying the same amounts of tuna and sprat separately do not get it. Though it seems that the discount is arbitrary, it is not so. She has well-formed formulae that make her business attractive and lucrative.

We have more evidences to prove that illiteracy is secondary when compared with innumeracy. Kumaratunga, the famous Sri Lankan writer who characterized the typical gullible folk in his work *Hathpana*, focuses on the innumeracy of the guy, instead on illiteracy. On one occasion, this guy Kirihami is given eight *panam* and sent to the market to buy pots. The vendor says a pot costs ten *thuttu.* Getting mad at the vendor, Kirihami says, "You cannot fool me, I got only eight *panam* and you take it and give the pot to me". What a deal!

When I walk down further, I find the small shop owned by the daughter of that old lady. Still in her thirties, she is courageous as her mother. This small place is being used to sew clothes, and she gets a fair income through the business. Her suppliers are from different places and they had different types of raw materials. So, selection of suppliers along with transportation becomes a major problem for her. Finally, the demand of her customers had to be met. When it comes to transportation, even the weather conditions and road conditions do matter. Thus, finding the optimal possibility would reduce her cost and make her business much beneficial. Though she is used to do this intuitively and naively, one day she may have to adopt the mathematical algorithms or software to make profits, if she expands her business beyond a certain level.

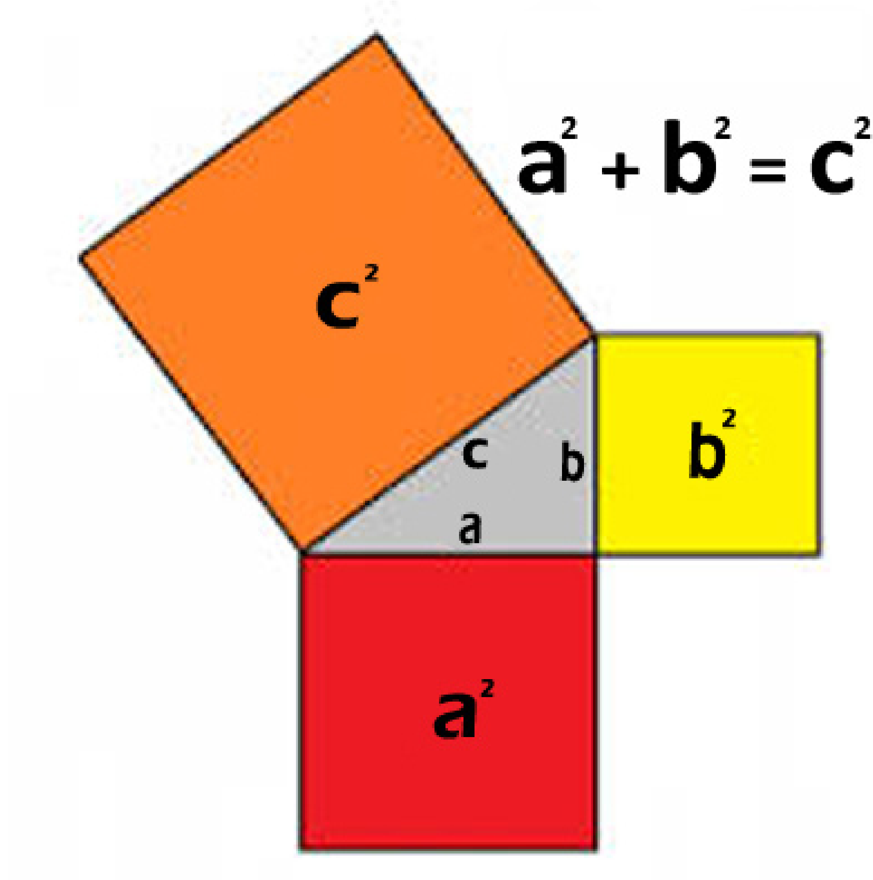
Mathematical and statistical modeling becomes vital in this case. For instance, she has previous records of road conditions, collected through her past visits to the suppliers. These records could be thought as data and statistical tests could be performed of them. Thus, we can get some precise measure on the road conditions. But road conditions make only one restriction. We can gather all such restrictions, and build a general mathematical model. The primary objectives are reducing the cost, and meeting the customer requirements. And in the core of the model is the selection of suppliers. This can be done through assignment algorithms in optimization, a fast developing branch of mathematics.

This depicts the nature of mathematical modeling. You encounter a problem in your real life. Then you take it out from the realistic norms and express in a mathematical realm in a symbolic language. It is no longer a real-life problem; you have a mathematical problem instead. Mathematical problems are subjected to mathematical treatment, and if a solution exists, you find it. Now this solution has to be transformed back to the domain of real life. Finally, you have solved the real-life problem.



The daughter of the old fishmonger shows signs of becoming a successful

Pythagoras theorem



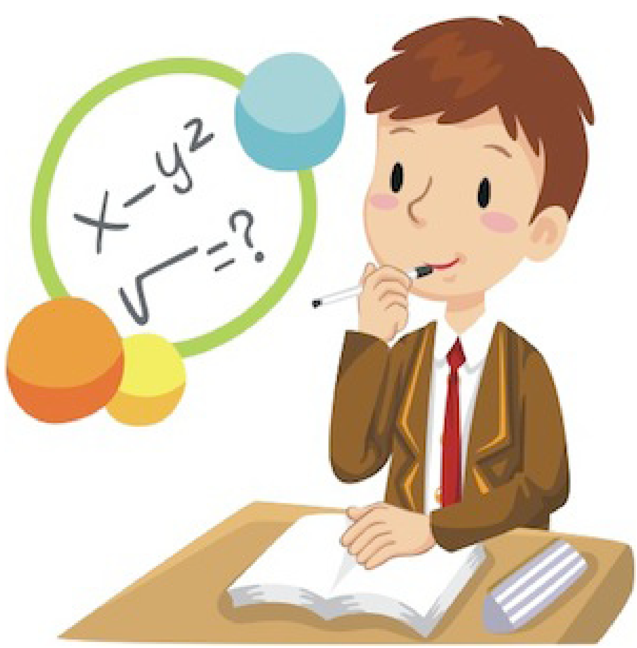
businesswoman in the future. She has already decided to expand her business by adding an annex to the current premises. Probably she is going to recruit more sewing-girls and get more orders. Presence of the mason and his crew at her place in the early morning indicates that the construction work of the annex is going on smoothly. It is somewhat amazing, because normally the mason and his crew arrive on time only on the day in which the very first stone is laid at an auspicious moment.

But no one dares to get across with the mason on this habitual delay. Everybody was aware of his bad temper and harsh words. Even now he shouts at someone in his crew in an incredible language of his own. Allegedly, the poor guy has not taken three-four-five accordingly. This is how they refer to the Pythagorean Theorem: three-four-five. The theorem says that, in a right angle triangle, the sum of the squares of the side lengths of the perpendicular sides is equal to the square of the diagonal. Most comprehensive example of this theorem is the right triangle with side lengths of three, four and five units. That is how the term has been coined by people who knew the theorem by its application. Though the mason may have never heard about Pythagoras, he is not only entrusted with the application of the Pythagorean Theorem, but also loyal enough to it, as to respond rigorously when it is being misused by somebody.

Historical evidence prove that the theorem which is attributed to Pythagoras had been known even before Pythagoras, in Babylonian, Chinese and Indus civilizations. This depicts the remarkable co-existence of mathematics and day to day life. Millions of day to day lives together make the history of mankind. Man invented (or rather discovered, it is another chicken-or-egg dilemma!) mathematics from the day to day experience of life. They may have observed this relationship of side lengths of right triangles, probably during some construction or measurement work. Numerous people like our mason may have benefitted from that observation. A small minority among them like Pythagoras may have pondered about proving the observation. Though a proof does not make any sense to the mason who always verifies something by application and intuition, for the people who bother about deriving further results or abstractions, a proof marks a milestone. At one time in history, the task of creating knowledge was transferred from people like our mason, to people like Pythagoras. This originated a well-organized system of postulates, results and proofs, and it was called mathematics. This theorem, which is an element of this system, is named not after the masons who first observed or applied it, but Pythagoras, who seemingly gave the first-ever acceptable proof. One would say that the mason just applies the theorem of Pythagoras, yet he got it not from Pythagoras, but his master, the late mason. The great masters observed it through their day to day life experiences.

As mentioned early, well-organizing of such observations gave birth to the system we call mathematics. The basic principles used in the organizing procedure lay the foundation. It is nothing else, but mathematical logic. If you work-out a mathematical problem, or prove a theorem, then you are supposed to be logical. Loosely, you should make sure that you are not violating the logical principles. You need to be logical, or rather rational in your day to day life as well. So, we are using so much logic in our day to day conversations, though unintentionally.

The most important logical principle in mathematics must be the rule of implication. Descarte, the famous mathematician and logician, Carrol, the famous writer, and Russel, the famous philosopher were masters of this rule. However, I found that even my master, the chief at my office had mastered the rule. I am just reminded of his face, as I see the time. Four minutes late than usual! I wasted two minutes on looking for the socks, one minute for the breakfast of the cat, and slowed down at the old fishmonger, her daughter, the mason and the crew. This may have taken an extra minute. Though it is only four minutes right now, it will grow during the trip. The variation of the traffic jam with time is non-linear. It is nearly exponential. That means, I would be considerably late today.



Being late is never tolerated in work. Our boss introduced an effective method to avoid it. He informed that, if somebody is late for more than four days a month, he or she would not be given a bonus at the end of that month. Last month, I was late only on two days. Yet I received no bonus at all. There is nothing wrong with the boss. He simply used the rule of implication. Suppose a statement implies another statement. Whenever it is true, it must be true. But false does not require to be true. For instance, for a real number, the implication *if, then* is obviously true. Whenever a value is substituted for satisfying, also is satisfied. But for the selection, the statement is false, but the statement is true. Similarly, for the selection of only two late records, the statement *I was late more than four days* becomes false, but *I would not be granted any bonus* can be true. The implication is not violated.

The strategy was logic. It is the same strategy used by mankind to organize and systematize their observations. Mathematics is such a system. These observations were called *mathematical principles* within that system. Yet they can never be separated from their origin: the day to day life. That is why they are being reflected from day to day activities. It's not incorrect to call this an application of mathematical principles. Yet these principles originated from day to day activities themselves and not through pencils and papers. It is no wonder that the day to day life is full of mathematical elements. We must be amazed, if there was any day to day life with no mathematical elements, or mathematics with no day to day applications.

Anuradha Mahasinghe
Department of Mathematics
University of Colombo

Mathematical patterns hidden in nature

Dr Deepal Subasinghe

Do you know that nature is an excellent mathematician?

When we hear the term mathematics do not most of us think of numerals, algebraic equations, geometrical shapes, graphs etc.? But do you know that in nature the patterns in beautiful flowers, shells, fruits and the patterns on the skins and other coverings of animals are all arranged according to certain mathematical principles? Why is the shell of a snail or the shell of a sea snail twisted in a spiral manner? When such organisms grow bigger how do they still retain this shape?

Let us consider this first. Even though it is easy to recognise the patterns in nature' to understand them is not easy. Biologists study the patterns in plants, flowers, fruits and animals and try to give biological explanations and the evolutionary basis for these phenomena.

However most people are unaware or forget that these patterns in living organisms as well as in some non - living things are based on mathematical principles.

Life is involved in a continuous struggle for its survival.

In this struggle the easiest pathway has to be selected to reach its goal while being subjected to the effects of various factors in nature and the environment, and also those exerted by others of its own species around it.

In such situations some of the resulting patterns may be quite complex while others may be quite obvious.

As an example let us consider a simple arithmatic series.

The series 2,4,6,8,10......... is formed by the addition of 2 to the earlier number. It is possible to form such series using some other mathematical method and some of these patterns may be more attractive or fascinating than others.

The following series developed by Fibonacci creates very beautiful patterns.
Here the mathematical method is to begin with 0 and 1 and add the two together to from the next number.
Example:

0+1 = 1
1+1 = 2
1+2 = 3
2+3 = 5
3+5 = 8
5+8 = 13
8+13 = 21
............and so on.

Then the Fibonacci series of 0,1,1,2,3,5,8,13,21,34 etc. is formed.
The next time you visit a flower garden or a home garden with plants bearing flowers count the number of petals in various flowers.

You will notice that most flowers will have either 5 or 8 petals or more.

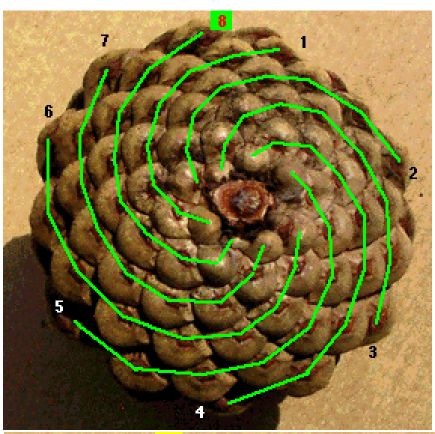
Also you will notice that flowers with four (4) petals are rare. Some flowers have 6 petals and they are mostly composed of 3 pairs of two each. Number 4 does not belong to the Fibonacci series.
If you are living in the hill country or visit a place with a pine tree'

Image 1

Image 2

Image 3

examine a cone of Pinus (Images 4 and 5). Observe carefully the arrangement of their "petals" (sporophylls), and you will see clearly that they are arranged in a spiral manner. When you look more carefully you will be able to see that anti clock wise (turning left) as well as clock wise (turning right)spirals exist. If you count these spiral lines you will see that they will be two consecutive numbers of the Fibonacci series.



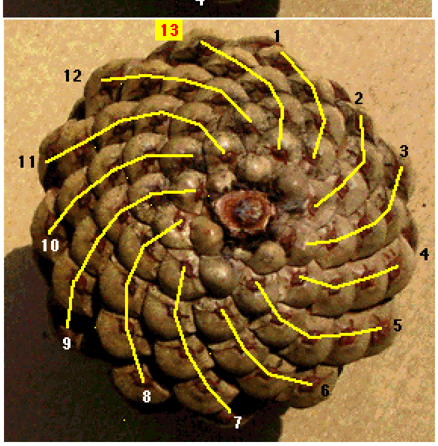


Image 4 and Image 5

Also if you study the distance between the branching points of certain plants you will be able to observe the Fibonacci series.

Consider a young plant after it is newly grown (germinated). It is compulsory for its stem to be strong before it branches out.
Let us assume that it takes two months for this to happen.

If the first branching is after two months and the subsequent branching takes place once a month' then the pattern would be as shown in the figure. A good example for this is the small plant with the scientific name *Achillea Ptarmica*. If you study the environment carefully you will be able to identify a large number of plants with similar patterns.

If we consider the population in a colony of bees you can observe another hidden Fibonacci pattern. We know that a bee colony has a queen, 'worker' females and a few male bees. All female and male bees are the offspring of the queen bee. The female bees are born from the fertilized eggs. Now let us consider the geneology or the family tree of a male bee.

The male bee has only a mother (queen bee). That is only one parent (1). However it has a grand father and a grand mother (The mother and the father of the queen bee) (2). It has no great grand father because its grand father has no father.

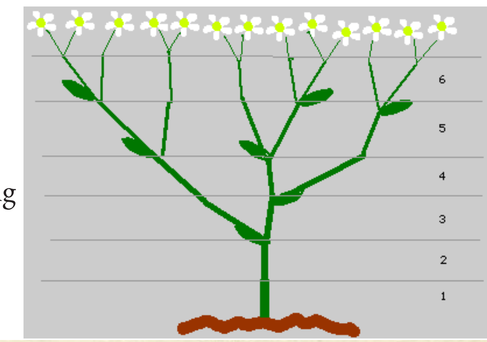


Image 6 and Image 7

It is clear that its series is 1,2,3,5,8,.............

Now let us build a Fibonacci spiral for which we require a graph paper with 13 × 21 squares (Image 8). Now make 1 which is the first number in the (9,6) sector of the graph paper (as shown in the following diagram) Now draw an arc from the top end of the first rectangle to its lower diagonal and extend it to the diagonal of the rectangle on its left. Then extend the arc to the top diagonal of the 2 × 2 rectangle aboue it. Now draw this arc as an unbroken line through to the 3 × 3 rectangle to the 5 × 5 rectangle, 8 × 8 rectangle and finally to the 13 × 13 rectangle. Then you will have a trace as shown below as Image 9. It is a Fibonacci spiral.

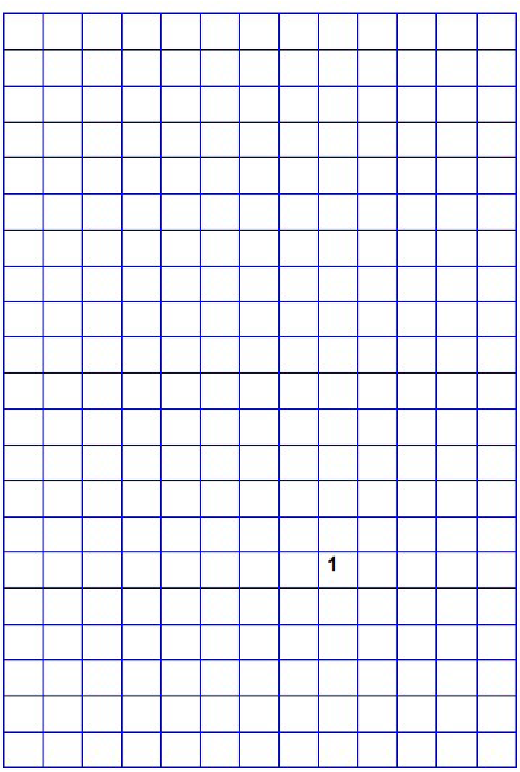


Image 8

Can you see this spiral in the Images 10 to 13.

Now let us observe the number in the Fibonacci series.

1,3,5,8,13,21,34 ....Let us consider the ratio of these numbers with their consecutive number.

2 ÷ 1 = 2
3 ÷ 2 = 1.5
5 ÷ 3 = 1.666...
8 ÷ 5 = 1.6
13 ÷ 8 = 1.625
21 ÷ 13= 1.615
34÷ 25 = 1.619

You will notice that this ratio gradually approaches l.61803. You will notice that the ratio between the number of left handed (anti clock wise) line and the right handed (clock wise) line in the pinus cone is similar.

This is the 'golden ratio' of the Fibonacci series.

You have to do a similar thing using the calculator in your computer or the cellular phone.

First enter number 1 and press the button 1/x
Add number 1 and press 1/x again
Add number 1 and press 1/x again

Continue to do this until the number you see on the calculator screen does not change any more. You will see that the number that does not change is 1.61803.

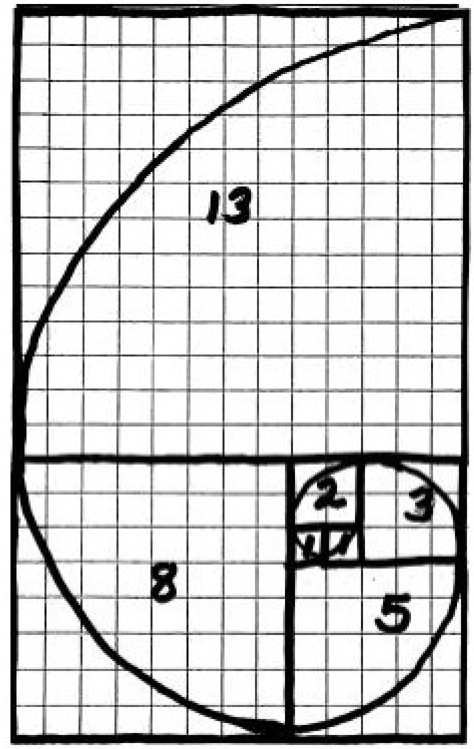


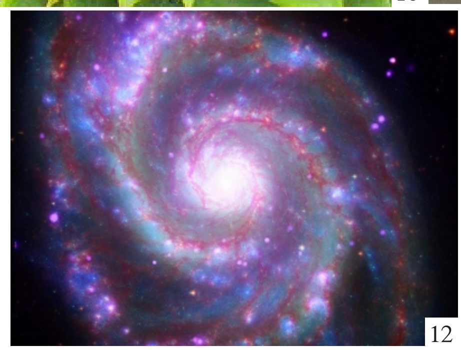
Image 9

How do plants, flowers, fruits or the outer coverings of animals get such a ratio or a series. It is possible to give various scientific answers for this observation. For example if the leaves in a plant are arranged in a spiral manner the upper leaves do not prevent the sunlight reaching the lower leaves. Therefore all leaves receive a certain amount of sunlight and the young leaves receive more sunlight and produce a greater amount of food.

Now let us consider another simple shape that is found in nature. It is no secret that objects in nature starting from atoms, small cells to planets and stars in the universe have a circular or spherical shape. Let us consider a cell first. Let us consider (assume) a two - dimensional cell for convenience. That is, a flat cell on a plane which is a living cell that uses water and food and tries to increase


10


11


12

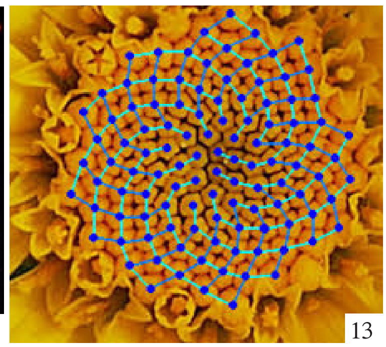

13

Image 10, Image 11, Image 12 and Image 13

Table 1:

Shape	Number of sides	Perimeter cm	Area Cm^2	Perimeter/ area
Equilateral triangle	3	45.5901	100	0.456
Square	4	40.0	100	0.4
Hexagon	6	37.22	100	0.372
Circle	Infinite	35.449	100	0.354

Table 2:

Shape	Number of facets	Volume	Surface area
Tetrahedron (Pyramidal)	4	1	7.21
Cube	6	1	6
Octahedron (Double Pyramid)	8	1	5.72
Sphere	Infinite	1	4.84

its size. But a problem faced by the cell is its tendency to lose water though its cell wall. The cell should minimize its perimeter or the length of its cell walls to minimize this loss. In order to achieve this the cell will minimize its perimeter while its area is maximized.

Let us think that the cell has an option of selecting one of the simple shapes of a circle, triangle, square and a hexagon. Let us consider a shape which has an area of $100cm^2$ for comparison.

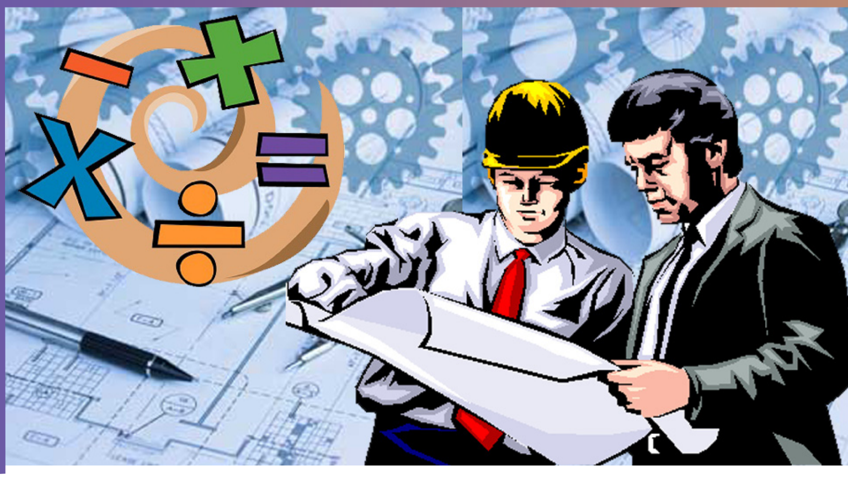
As can be seen the ratio between the perimeter and the area is minimum in a circle. Now let us consider these dimensional shapes. In this instance the surface area is important rather than the perimeter.

It is clear that the spherical object has the minimum surface area but the maximum volume. The drops of water falling from a height (ex. raindrops), the planets and soap bubbles are spherical because they are trying to reduce their surface area. This is why the soap bubbles you blowout assume only a spherical shape irrespective of the shape of the frame you use.

Dr Deepal Subasinghe
Senior Scientist
Institute of Fundamental Studies
Hantana Road
Kandy

Mathematics in Engineering

Prof. Priyan Dias



Introduction: Is Engineering "Mathematics in Disguise"?

Engineering is sometimes called "Mathematics in disguise", judging from what is taught in most engineering faculties. Such faculties in most parts of the world also want potential entrants to demonstrate good skills in mathematics at secondary school level. Nevertheless, an engineer is very different from a pure mathematician, and some of these differences are also explored in this article.

So what are the kinds of mathematics used by engineers? In fact a wide variety is used, ranging from simple linear equations to calculus and statistics. Someone has defined engineering as "the study of responses of objects and systems to perturbations, in the time, space and frequency domains."

Mechanics

The mechanics based engineering disciplines (such as civil and mechanical engineering) rely heavily on Newton's Laws, whether to describe equilibrium or motion. So, although a body is in equilibrium only if at rest or moving with uniform velocity, an equilibrium equation can be written even for a mass (m) with acceleration (a), by considering the term "ma" as a force, giving rise for example to the equation describing simple harmonic motion:

$$ma = -kx,$$

where k is a spring constant, x the displacement from an equilibrium position and the term "kx" a restoring force (see Figure 1).

A lot of engineering has to deal with non-linear behaviour. For this reason, differential equations are the "bread and butter" of engineers. In the simple harmonic equation described above, the acceleration is not constant, but changes with time. Hence, we write acceleration as dv/dt (or change in velocity within an infinitesimally small time dt), and similarly velocity as dx/dt. This will cause the above simple harmonic motion equation to be written as

$$m (d2x/dt2) = -kx,$$

and this is called a differential equation, which if solved will give us an expression for how x changes with t.

In the above problem, we considered only a point mass. However, when we need to solve problems involving

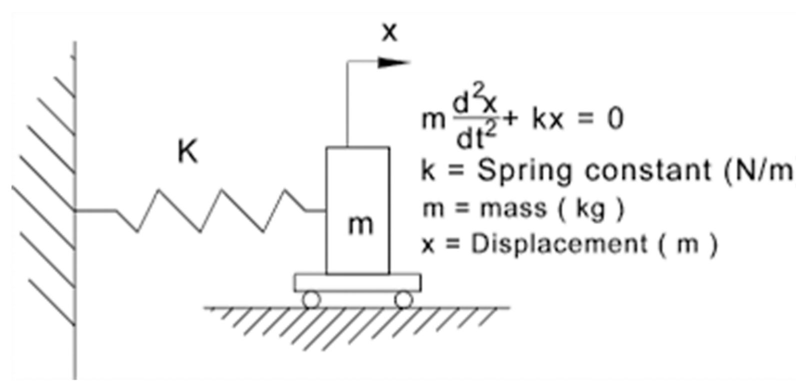


Figure 1 – The mathematics of simple harmonic motion

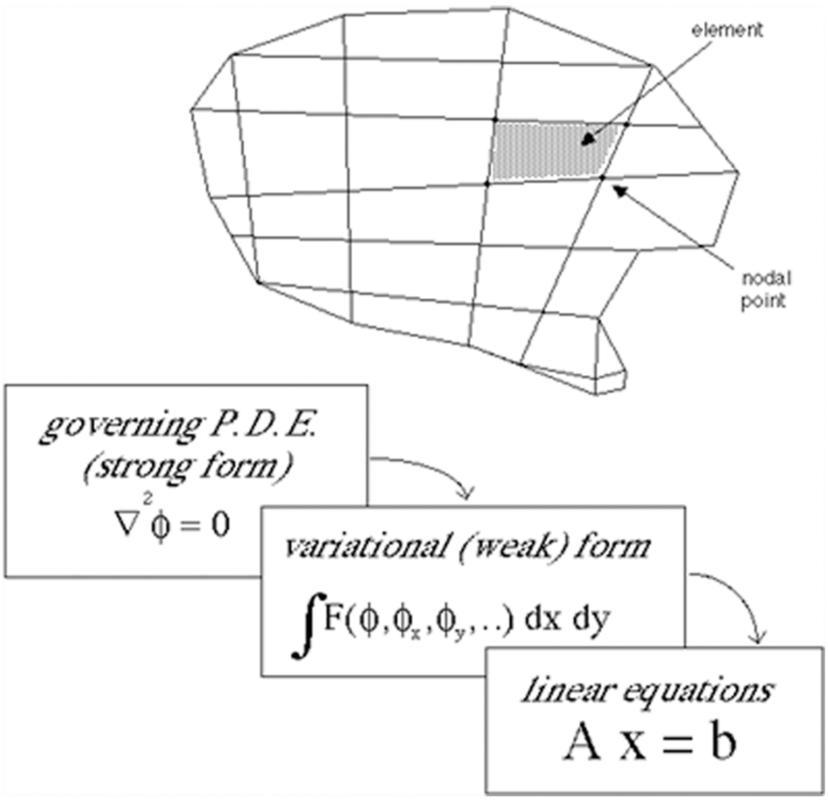


Figure 2 – Converting partial differential equations to linear simultaneous equations via finite elements

objects with complex shapes or many parts, we often resort to the finite element method. Here the object is divided (or "meshed") into small "finite elements" (see Figure 2), within which displacements can be represented by simple functions that approximate the actual displacement field. We can then get expressions for strains from displacements, link stresses to strains using known elastic properties (such as Hooke's Law and Poisson's ratio), and relate the stresses to the forces applied. This way, we can find the displacements caused by a system of forces on a complex body. All the differential equations involved in this procedure are in fact converted to a set of equivalent simultaneous equations. Such simultaneous equations are solved using matrix methods, in particular matrix inversion.

"Electrics"

The notion of fields is very important in electrical engineering. For example Maxwell's equations describe electrical and magnetic fields and the relation between them. The electric motor and electricity generator are common applications of this relation between electric and magnetic fields.

Complex numbers is another branch of mathematics that is used in electrical engineering. Complex numbers have both real and imaginary components (see Figure 3), and these components are orthogonal (i.e. perpendicular) to each other. This is useful in describing the impedance of an alternating current (ac) electric circuit, which can have resistance, impedance and capacitive components. Resistance is considered the real component and the other two combined (called reactance) the imaginary one. The complex number mathematics helps us to understand why the reactance components cause a phase lag between voltage and current.

Most of the engineering entities described above (whether forces on masses or voltages in ac circuits) have direction and are hence vectors. For that reason the mathematics of vectors, and concepts such as vector products and direction cosines are used frequently in engineering. The cross product of two vectors is another vector that is at right angles to both the original vectors (see Figure 4). It can be used to describe the Lorentz force experienced by a moving electrical charge in a magnetic field (referred to above). The dot product of two vectors is a scalar, as in the case of the work done by a force vector in

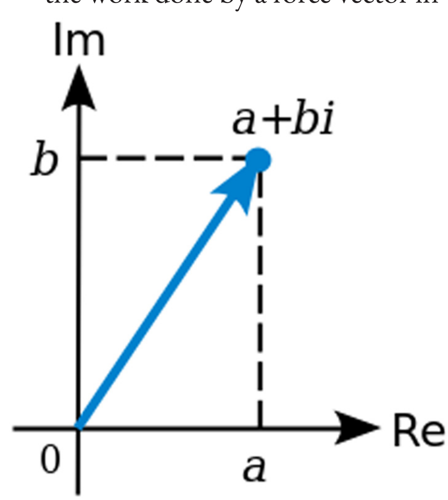


Figure 3 – Complex numbers have both Real (Re) and Imaginary (Im) components

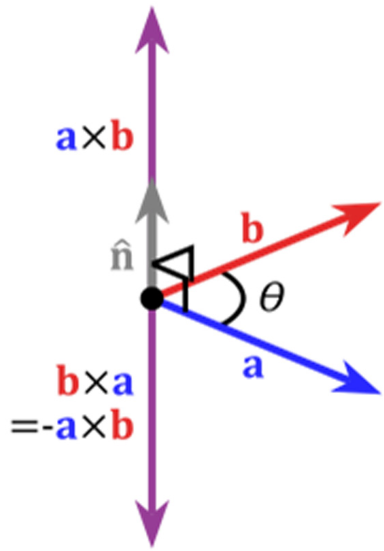


Figure 4 – The vector cross product is another vector

conjunction with a displacement vector.

Statistics

Among other things, statistics teaches us that entities can have not merely values but distributions of those values about a mean. This captures the notion of randomness, which is one aspect of uncertainty. Since engineering is carried out in an uncertain world, the mathematics of uncertainty is of great use in the practice of engineering. Figure 5 shows two normal distributions, characterized by means (μ) and standard deviations, for a load (Q) on say a structural element with resistance (R) of that element. The nature of these curves is that their "tails" are only asymptotic to the horizontal axis. There can always be a load higher than one encountered before, or a resistance lower. However, in order for structural design to be economical, we cannot design the element in a way that the highest possible load is less than the lowest possible resistance. We allow a certain overlap that is governed by an allowable probability of failure; and statistics can help us to compute this, using probability density functions such as those in Figure 5. For structural design this probability of failure is of the order of 1 in 106.

Statistics is also used in quality control, and is very important for any kind of production engineering. It can tell us how frequently tests need to be carried out and whether an unusual test result falls within the acceptable tolerance or outside it. It can guide us in making judgments about the entire population of data from results obtained for very limited samples of that population.

Design

We have hinted above about the idea of design, which is central to engineering. Design is a synthetic exercise, meaning that we try to create something, given certain requirements. Synthesis is in some ways the opposite of analysis, which involves investigating known entities. If we continue with the example of load and resistance in structural elements, the analysis task would be to find the stresses or deflections in the element, if given the loads on the element and the properties that contribute to its resistance. The synthesis task on the other hand would be to propose the properties of the element (e.g. type of material, dimensions of cross section) in order to carry the loads without exceeding certain stress and deflection limits. This is a much more difficult problem, sometimes called an "inverse" problem. Such inverse problems do not have unique solutions (unlike analysis problems). There can be more than one feasible solution. As such design can be considereda part of a wider activity called "search", i.e. search for the best solution.

Optimization problems are another type of search problem. Very often such problems are cast as minimization problems, because we seek minimum cost or minimum weight designs. Such problems also involve constraints that confine the solution to a given combination of parameters (or variables). Figure 6 is a representation of a linear function, f(x,y) that has to be minimized subject to linear constraints. The shaded area denotes the region that is allowed by the constraints and the minimum value of the function is given by one of the vertices of the triangular feasible region.

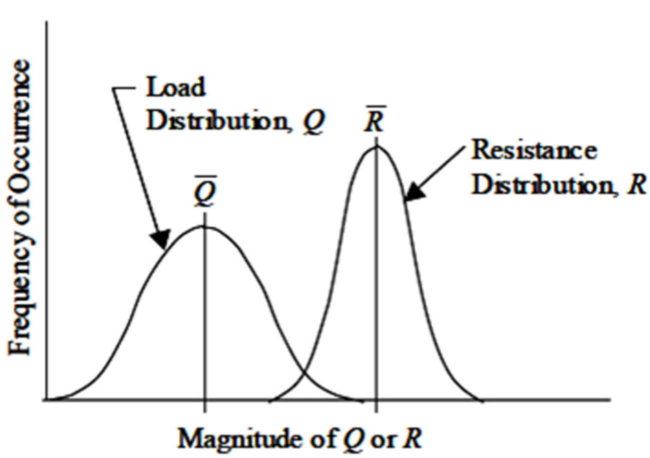


Figure 5 – Probability density functions for load and resistance

Practice

Although mathematics is so clearly central to engineering, it is only one among many tools used by an engineer. Furthermore, engineers

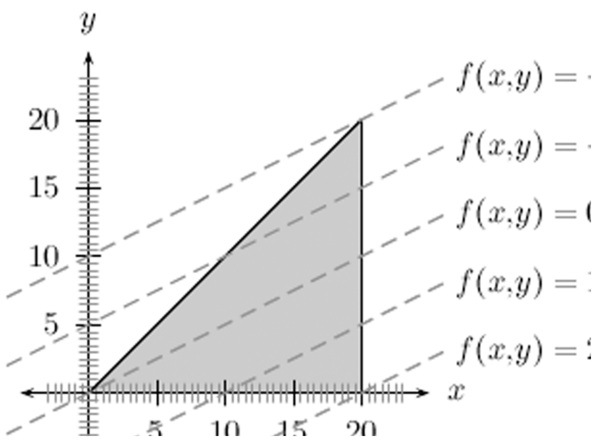


Figure 6 – Representation of a linear function subject to linear constraints

use mathematics in a very practical way, as opposed to a theoretical one. There is a story about a mathematician and an engineer who were placed at one end of a small room, with a cake at the other end. They are told that they can cross the room and take the cake. However, after one step is taken to cross the room, the next step to be taken can only be half as long as the first one, the third step half as long as the second and so on. The mathematician thinks about this and discovers that he can (theoretically) never reach the cake. The engineer however takes a few steps, gets close to the cake (because the room is small anyway), and reaches out to take the cake saying "close enough for all practical purposes". So engineers use clever approximations to arrive at practical solutions.

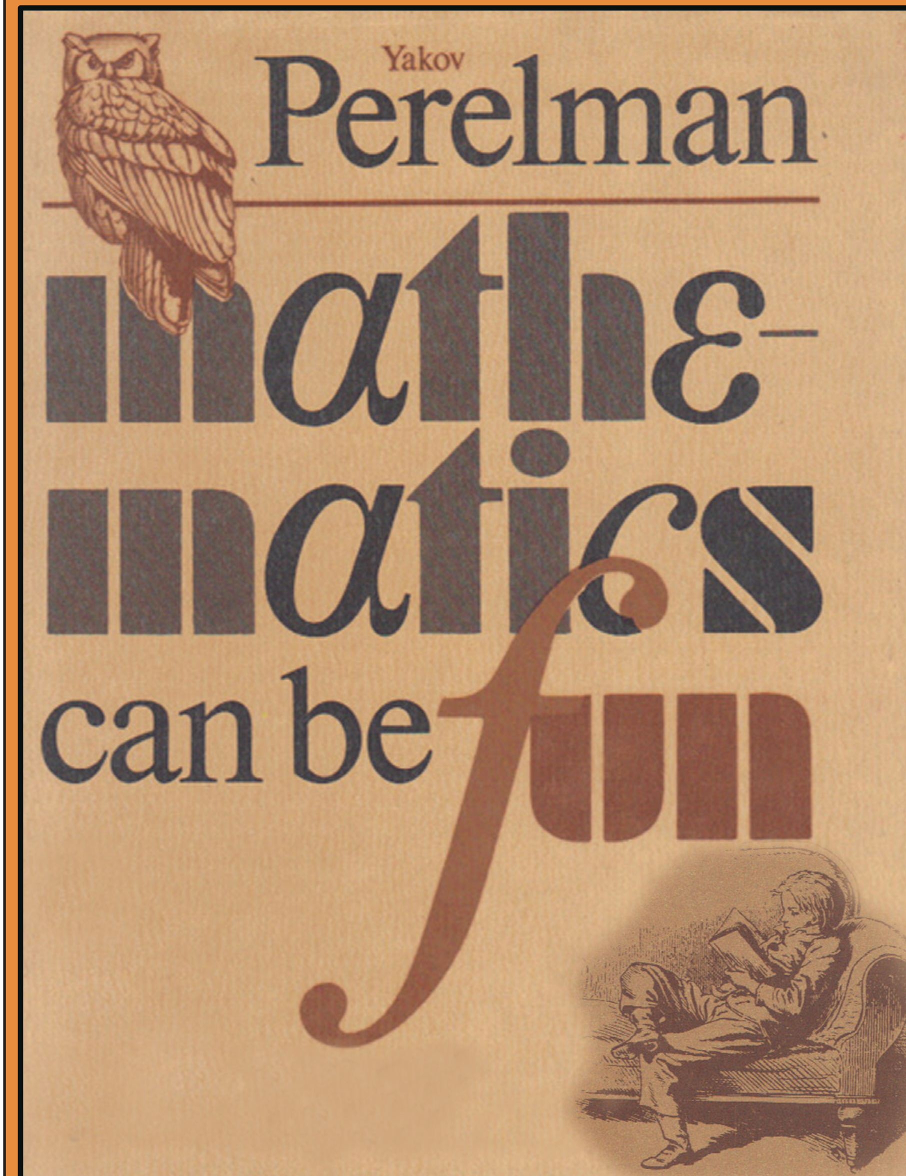
This need for approximation arises from what is called "idealization" in engineering. Engineers deal with a large number of objects and systems in the real world. These can range from skyscrapers and spacecraft through the network of wires that bring us electricity to mobile phones and digital cameras. It is said that humans are perhaps the only animals who build things twice. The first time round, they build things in their minds – this is probably called planning in the everyday world. During this stage, all the problems that can affect the final product are thought of and dealt with in our minds. It is only after this that the object or system is created in the real world. The "planning" done by engineers is sometimes called "design" and often called "modelling". Mathematics is very prominent in such models, which nowadays are almost always computer based. In constructing such models however, not all aspects of the real world object or problem can be represented mathematically, and approximations have to be made. This is what is called idealization. Idealization is done by erring on the side of caution, hence engineering models are said to be conservative. Accuracy is not as important for engineers as safety. It is better to have a somewhat inaccurate model that will result in a safe design, rather than a more accurate model with a lower margin of safety. These are the kinds of decisions that engineers have to make about their mathematical models.

Conclusion: Mathematics is only a part of Engineering

Mathematical models are not the only basis for design. One reason for this is that engineering artifacts have been designed and built very much before the use of mathematics in engineering. Egypt's pyramids, Rome's bridges and Sri Lanka's irrigation systems are some examples. This body of knowledge that has arisen from practice is often codified as "thumb rules". Engineers therefore will sometimes use these thumb rules to arrive at a preliminary design before using mathematical tools to refine it.

Another reason is that there are some aspects of a design that cannot be quantified, but may still be very important. For example, it may be possible, using mathematical tools, to re-design the trace of a roadway so that high speeds and reduced travel time can be achieved. However, if it is not possible to relocate the retail shops along that road, it may be that those high speeds will be unsafe for the large number of pedestrians that will be using the sidewalks. This is why Einstein is supposed to have said that "not everything that is countable counts; and not everything that counts is countable." Thinking in this way is called "reflective practice" or "systems thinking", something that all professionals are called upon to do. This is especially important for engineers, who will often be tempted to pay "selective inattention" to any aspect of a problem that cannot be represented in a mathematical model.

Prof. Priyan Dias
Department of Civil Engineering
University of Moratuwa



About the Author

In 1913 in Russian bookshops appeared a book by the outstanding educationalist Yakou Isidorovich Perelman entitled Physics for Entertainment. It struck the fancy of the young who found in it the answers to many of the questions that interested them.
Physics for Entertainment not only had an interesting layout, it was also immensely instructive.
In the preface to the 11th edition Perelman wrote: "The main objective of Physics for Entertainment is to arouse the activity of scientific imagination, to teach the reader to think in the spirit of the science of physics and to create in his mind a wide variety of associations of physical knowledge with the widely differing facts of life, with all that he normally comes in contact with."
Physics for Entertainment was a best seller.
Y. I. Perelman was born in 1882 in the town of Byelostok (now in Poland). In 1909 he obtained a diploma of forester from the St. Petersburg Forestry Institute.
After the success of Physics for Entertainment Perelman set out to produce other books, in which he showed himself to be an imaginative popularizer of science. Especially popular were Arithmetics for Entertainment, Mechanics for Entertainment, Geometry for Entertainment, Astronomy for Entertainment, Figures for Fun, Physics Everywhere, and Tricks and Amusements. Today these books are known to every educated person in the Soviet Union.
He has also written several books on interplanetary travel (Interplanetary Journeys, On a Rocket to Stars, World Expanses, etc.).
The great scientist K. E. T siolkovsky thought highly of the talent and creative genius of Perelman. He wrote of him in the preface to Interplanetary Journeys: "The author has long been known by his popular, witty and quite scientific works on physics, astronomy and mathematics, which are moreover written in a marvelous language and are very readable."
Perelman has also authored a number of textbooks and articles in Soviet popular science magazines.
In addition to his educational, scientific and literary activities, he has also devoted much time to editing. So he was the editor of the magazines Nature and People and In the Workshop of Nature.
Perelman died on March 16, 1942, in Leningrad.
Many generations of readers have enjoyed Perelman's fascinating books, and they will undoubtedly be of interest for generations to come.

Preface

To read and enjoy this book written by Ya. Perelman will suffice to possess a modest knowledge of mathematics, i.e. A knowledge of the rules of arithmetic and elementary geometry. Very few problems require the ability of forming and solving equations, and the simplest at that.

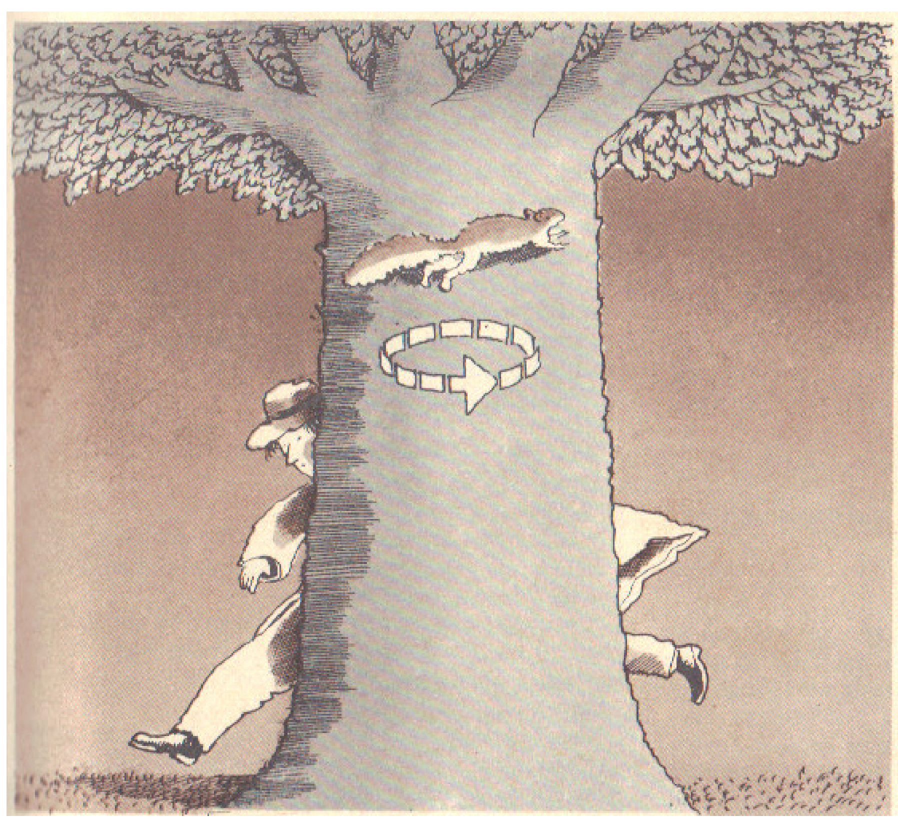
The coverage may be, is quite diverse: the subject ranges from a motley collection of conundrums and mathematical stunts to useful practical problems on counting and measuring. The author has done everything to make his book as fresh as possible, avoiding repetition of all that has already appeared in his other works (Tricks and Amusements, Interesting Problems, etc.). The reader will find a hundred or so brain-teasers that have not been included in earlier books. Chapter 6 - "Number Giants" - is adapted from one of the author's earlier pamphlets, with four new stories added.

01. Brain - Teasers for Lunch

It was raining...... We had just sat down for lunch at our holiday home when one of the guests asked us whether we would like to hear what had happened to him in the morning.
Everyone assented, and he began.

1. A Squirrel in the Glade

"I had quite a bit of fun playing hide-and-seek with a squirrel," he said. "You know that little round glade with a lone birch in the center? It was on this tree that a squirrel was hiding from me. As I emerged from a thicket, I saw its snout and two bright little eyes peeping from behind the trunk. I wanted to see the little animal, so I started circling round along the edge of the glade, mindful of keeping the distance in order not to scare it. I did four rounds, but the little cheat kept backing away from me, eyeing me suspiciously from behind the tree. Try as I did, I just could not see its back."
"But you have just said yourself that you circled round the tree four times" one of the listeners interjected.
"Round the tree, yes, but not round the squirrel."
"But the squirrel was on the tree, wasn't it?"
"So it was."
"Well, that means you circled round the squirrel too."
"Call that circling round the squirrel when I didn't see its back?"
"What has its back to do with the whole thing? The squirrel was on the tree in the centre of the glade and you circled round the tree. In other words, you circled round the squirrel."
"Oh no, I didn't. Let us assume that I'm circling round you and you keep turning, showing me just your face. Call that circling round you?"
"Of course, what else can you call it?"



"You mean I'm circling round you though I'm never behind you and never see your back?"

"Forget the back! You're circling round me and that's what counts. What has the back to do with it?"

"Wait. Tell me, what's circling round anything? The way I understand it, it's moving in such a manner so as to see the object I'm moving around from all sides. Am I right, professor?" He turned to an old man at our table.

"Your whole argument is essentially on about a word," the professor replied. "What you should do first is agree on the definition of 'circling'. How do you understand the words 'circle round an object'? There are two ways of understanding that. First, it's moving round an object that is in the centre of a circle. Secondly, it's moving round an object in such a way as to see all its sides. If you insist on the first meaning, then you walked round the squirrel four times. If it's the second that you hold to, then you did not walk round it at all. There's really no ground for an argument here, that is, if you two speak the same language and understand words in the same way."

"All right, I grant there are two meanings. But which is the correct one?"

"That's not the way to put the question. You can agree about anything. The question is, which of the two meanings is the more generally accepted? In my opinion, it's the first, and here's why. The sun, as you know, does a complete revolution in a little more than 25 days....."

"Dose the sun revolve?"

Of course, it does, like the earth around its axis. Just imagine, for instance, that it would take not 25 days, but 365 1/4 days, i. e. a whole year, to do so. If this were the case, the earth would see only one side of the sun, that is, only its 'face'. And yet, can anyone claim that the earth does not revolve round the sun?"

"Yes, now it's clear that I circled round the squirrel after all."

"I've a suggestion, comrades!" one of the company shouted. "It's raining now, no one is going out, so let's play riddles. The squirrel riddle was a good beginning. Let each think of some brain-teaser."

"I give up if they have anything to do with algebra or geometry." a young woman said.

"Me too," another joined in.

"No, we must all play, but we'll promise to refrain from any algebraical or geometrical formulas, except, perhaps, the most elementary ones. Any objections?"

"None!" the others chorussed.

"Let's go."

"One more thing. Let the professor be our judge."

2. School - Groups

"We have five extra-curricular groups at school," a schoolboy began. "They're fitters', joiners', photographic, chess, and choral groups. The fitters group meets every other day, the joiners' every third day, the photographic every fourth day, the chess every fifth day and the choral every sixth day. These five groups first met on January 1 and thenceforth meetings were held according to schedule. The question is, how many times did all five groups meet on the same day in the first quarter (January 1 excluded)?"

"Was it a Leap Year?"

"No."

"In other words, there were 90 days in the first quarter."

"Right."

"Let me add another question," the professor broke in. "How many days were there when none of the groups met in the first quarter?"

"So there's a catch to it! There won't be another day when all the five groups meet, and there won't be one day when none meet. That's clear!"

"Why?"

"Don't know. But I've a feeling there's a catch."

"Comrades!" said the man who had suggested the game. "We won't reveal the results now. Let's have more time to think them. The professor will anounce the answers at supper."

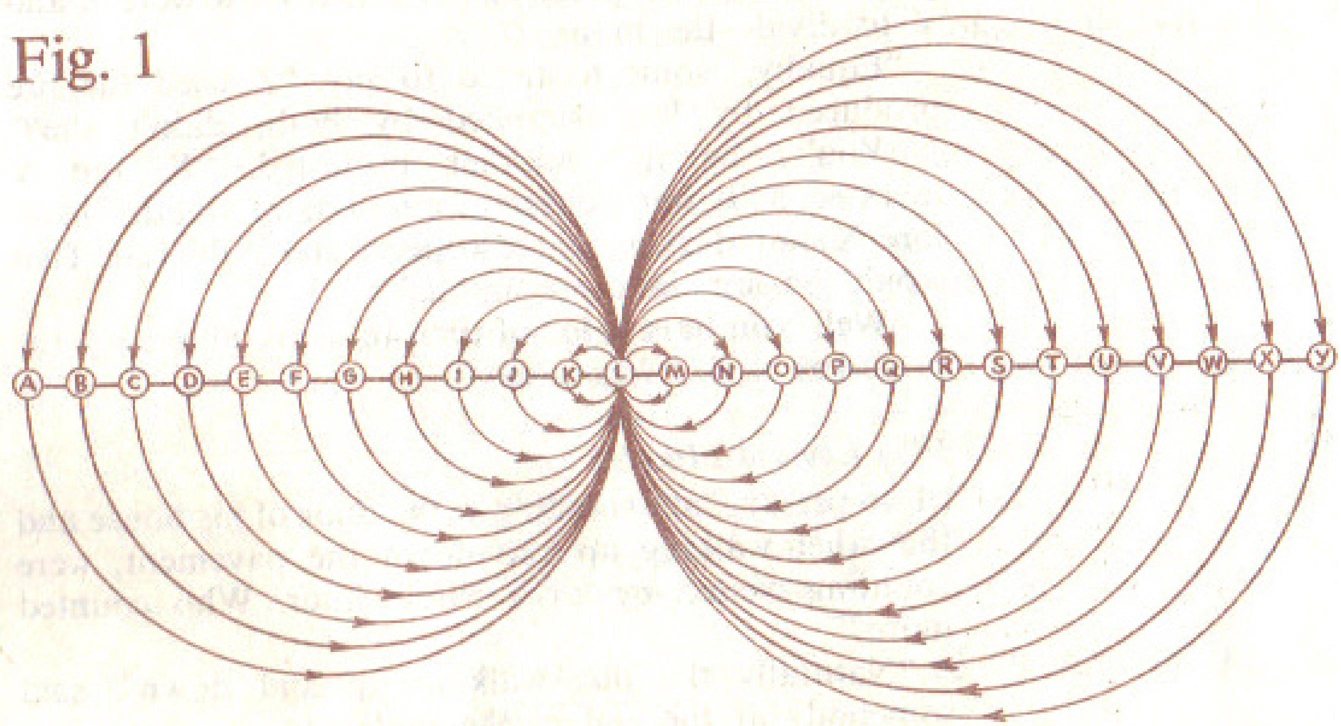
3. A Log Problem

"It happened at a summer cottage. A household problem, you might call it. The cottage is shared by three persons - let's call them X, Y, and Z. It's an old house with an old-fashioned cooking stove. X put three logs into the stove, Y added five and Z, who had no firewood, paid them eight kopecks as her share. How did X and Y divide the money?"

"Equally," some hastened to say. "Z used the fire produced by logs supplied by both, didn't she?"

"You're wrong," another protested. "X and Y invested, as it were, different amounts of wood. Therefore, X should receive

Fig. 1



three kopecks and Y the rest. That would be fair, in my opinion."

"Well, you have a lot of time to think about it," the professor said. "Who's next?"

4. Who Counted More?

"Two persons, one standing at the door of his house and the other walking up and down the pavement, were counting passers-by for a whole hour. Who counted more?"

"Naturally the one walking up and down," said somebody at the end of the table.

"We'll know the answer at supper," the professor said. "Next!"

5. Grandfather and Grandson

"In 1932 I was as old as the last two digits of my birth year. When I mentioned this interesting coincidence to my grandfather, he surprised me by saying that the same applied to him too. I thought that impossible ...."

"Of course that's impossible," a young woman said.

"Believe me, it's quite possible and grandfather proved it too. How old was each of us in 1932?"

6. Railway Tickets

"I'm a railway ticket seller," said the next person, a young lady. "people think this job is easy. They probably have no idea how many tickets one has to sell, even at a small station. There are 25 stations on my line and I have to sell different tickets for each section up and down the line. How many different kinds of tickets do you think I have at my station?"

"Your turn next," the professor said to a pilot.

7. A Helicopter's Flight

"A helicopter took off from Leningrad and flew north. After five hundred kilometres it turned and flew 500 kilometres east. After that it turned south and covered another 500 kilometres. Then it flew 500 kilometres west, and landed. The question is, where did it land: west, east, north, or south of Leningrad?"

"That's an easy one," someone said. "Five hundred steps forward, 500 to the right, 500 back, and 500 to the left, and you're naturally back where you'd started from!"

"Easy? Well than, where did the helicopter land?"

"In Leningrad, of course. Where else?"

"Wrong!"

"Then I don't understand."

"Yes, There's some catch to this puzzle," another joined in. "Didn't

the helicopter land in Leningrad?"
"Won't you repeat your problem?"
The pilot did. The listeners looked at each other. "All right," the professor said. "We have enough time to think about the answer. Let's have the next one now."

8. Shadow

"My puzzle," said the next man, "is also about a helicopter. What's broader, the helicopter or its perfect shadow?"
"Is that all?"
"It is."
"Well, then. The shadow is naturally broader than the helicopter: sun-rays spread fanlike, don't they?"
"I wouldn't say so," another interjected. "Sun-rays are parallel to each other and that being so, the helicopter and its shadow are of the same size."
"No, they aren't. Have you ever seen rays spreading from behind a cloud? If you have, you've probably noticed how much they spread. The shadow of the helicopter must be considerably bigger than the helicopter itself, just as the shadow of a cloud is bigger than the cloud itself."
Then why is it that people say that sun-rays are parallel to each other? Seamen, astronomers, for instance."
The Professor put a stop to the argument by asking the next person to go ahead with his conundrum.

Answers 1 to 8

1. The squirrel puzzle was explained earlier, so we'll pass on to the next.

2. We can easily answer the first question: how many times did all the five group meet on the same day in the first quarter (January 1 excluded) by finding the least common multiple of 2,3, 4, 5 and 6. That isn't difficult. It's 60. Therefore, the five will all meet again on the 61st day - the fitters' group after 30 two-day intervals, the joiners' after 20 three-day intervals, the photographic after 15 four-day intervals, the chess after 12 five-day intervals and the choral after 10 six-day intervals. In other words, they can meet on the same day only once in60 days. And since there are 90 days in the first quarter, it means there can be only every other day on which they will meet.
It is much more difficult to find the answer to the second question: how many days are there when none of the groups meet in the first quarter? To find that, it is necessary to write down all the numbers from 1 to 90 and then cross out all the days when the fitters' group meets: e.g. 1, 3, 5, 7, 9, etc. After that one must cross out the joiners' group's days: e.g. 4, 7, 10, etc. When all the photographic, chess and choral groups' days have also been crossed out the numbers that remain are the days when there is no group meeting.
Do that and you'll see that there are 24 such days-eight in January, i.e. 2, 8, 12, 14, 18, 20, 24 and 30, seven in February and nine in March

3.It is not right to think, as many do, that eight kopecks were paid for eight logs, a kopeck for a log. The money was paid for one-third of eight logs because the fire they produced was used equally by all three. Consequently, the eight logs were estimated to be worth 8 × 3, i.e. 24 kopecks, and the price of a log was therefore three kopecks.
It is now easy to see how much was due to each. Y's five logs were worth 15 kopecks and since she had used 8 kopecks worth of fire, she would have to receive 15 - 8, i. e. 7 kopecks. X would have to receive 9 kopecks, but if you subtracted the 8 kopecks due from her for using the stove, you would see that she had to receive 9-8, i. e. 1 kopeck.

4.Both of them counted the same number of passers-by. While the one who stood at the door counted all those who passed both ways, the one who was walking counted all the people he met going up and down the pavement.
There is another way of putting it. When the man who was walking and counting the passers-by returned for the first time to the man who was standing at the door, they had counted the same number of passers-by - all those passing by the standing man encountered the walking man either on the way there or back. And each time the one who was walking was returning to the one who was standing he counted the same number of passers-by. It was the same at the end of the hour, when they met for the last time and told each other the final number.
5.At first it may seem that the problem is incorrectly worded, that both grandfather and grandson are of the same age. We shall soon see that there is nothing wrong with the problem.

It is obvious that the grandson was born in the 20th century. Therefore, the first two digits of his birth year are 19 (the number of hundreds). The other two digits added to themselves equal 32. The number therefore is 16: the grandson was born in 1916 and in 1932 he was 16.
The grandfather, naturally, was born in the 19th century. Therefore, the first two digits of his birth year are 18. The remaining digits multiplied by 2 must equal 132. The number sought is half of 132, i. e. 66. The grandfather was born in 1866 and in 1932 he was 66.
Thus, in 1932 the grandson and the grandfather were each as old as the last two digits of their birth years.

6.At each of the 25 stations passengers can get tickets for any of the other 24. Therefore, the number of different tickets required is: 25 × 24 = 600. There may also be round-trip tickets. If so, the number doubles and there are then 1 200 different tickets.

7.There is nothing contradictory in this problem. The helicopter did not fly along the perimeter of a square. It should be borne in mind that the earth is round and that the meridians converge at the poles (Fig.2.). Therefore, when the helicopter flew the 500 kilometres along the parallel 500 kilometres north of Leningrad, it covered more degrees eastward than it did when it was returning along the Leningrad latitude. As a result, the helicopter completed its flight east of Leningrad.
How many kilometres away? That can be calculated. Figure 3 shows the route taken by the helicopter: ABCDE. N is the North Pole where meridians AB and DC meet. The helicopter first flew 500 kilometres northward, i. e. along meridian AN. Since the degree of a meridian is 111 lilometres long, the 500-kilometre-long arc of the meridian is equal to 500 ÷ 111 ≈ 405'. Leningrad lies on the 60th parallel. B, therefore, is on 600 + 640 5. The helicopter then flew eastward, i. e. Along the BC parallel, covering 500 kilometres. The length of one degree of this parallel may be calculated (or learned from tables); it is equal to 48 kilometres. Therefore, it is easy to determine how many degrees the helicopter covered in its eastward flight: 500 ÷ 48 ≈ 1004'. Continuing, the aircraft flew southward, i. e. along meridian CD, and, having covered 500 kilometres, returned to the Leningrad parallel. Thence the way lay westward, i.e. along DA; the 500 kilometres Of this way are obviously less than the distance between A and D. there are as many degrees in AD as in BC, i.e. 10°4'. But the length of 1° at the 60th parallel equals 55.5 kilometres. Therefore, the distance between A and D is equal to 55.5 × 10.4 = 577 kilometres. We see thus that the helicopter could not have very well landed in Leningrad: it landed 77 kilometres away, on Lake Ladoga.

8.In discussing this problem, the people in our story made several mistakes. It is wrong to say that the rays of the sun spread fanlike noticeably. The Earth is so small in comparison to its distance from

Fig. 2

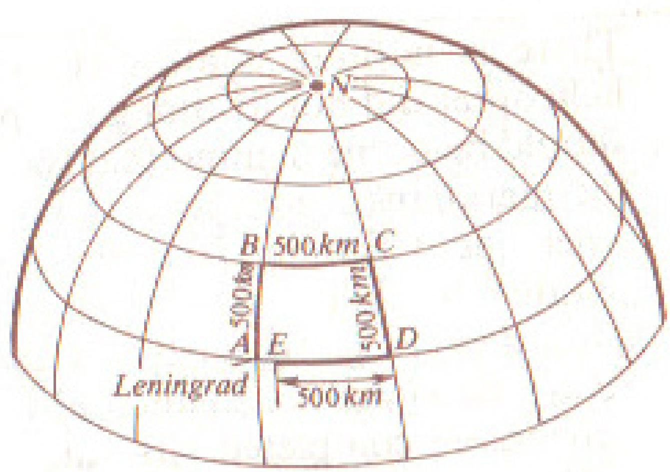
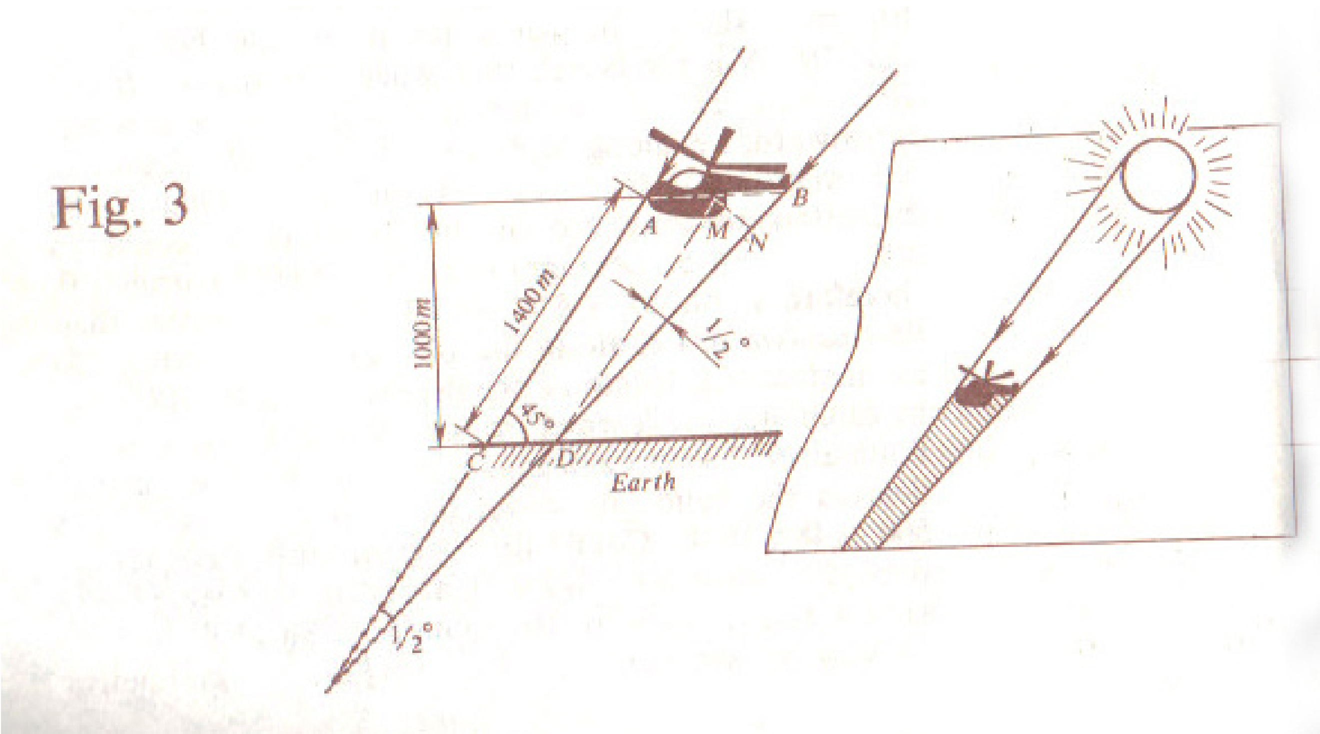


Fig. 3



the Sun that sun-rays falling on any part of its surface radiate at an almost absolutely imperceptible angle; in fact, rays may be said to be parallel to each other. We occasionally see them spreading fanlike (for instance, when the sun is behind a cloud). This, however, is nothing but a case of perspective.

Parallel lines, as they recede from the station point, always appear to the eye to meet far away at a point, e.g. railway tracks or a long avenue.

But the fact that sun-rays fall to the ground in parallel beams does not mean that the perfect shadow of a helicopter is as broad as the helicopter itself. Figure 3 shows that the perfect shadow of the helicopter narrows down in space on the way to the surface of the earth and that, consequently, the shadow the helicopter casts should be narrower than the helicopter, i.e. CD is shorter than AB.

It is quite possible to compute the difference, provided, of course, we know at what altitude the helicopter is flying. Let us assume that the altitude is 100 metres. The angle formed by lines AC and BC is equal to the angle from which the sun is seen from the earth.

We know that this angle is equal to 1/2°. On the other hand, we know that the distance between the eye and any object seen from an angle of 1/2° is equal to the length of 115 diameters of this object. Hence, section MN (the section seen from the surface of the earth at an angle of 1/2°) should be 1/115 of AC. Line AC is longer than the perpendicular distance between point A and the surface of the earth. If the angle formed by the sun's rays and the surface of the earth is equal to 45°, than AC (given the altitude of the helicopter is 100 metres) is approximately 140 metres long and section MN is consequently equal to 140 ÷ 115 = 1.2 metres. But then the difference between the helicopter and its shadow, i.e. section MB, is bigger than MN (1.4 times, to be exact), because angle MBD is almost equal to 45°. Therefore, MB is equal to 1.2 × 1.4, and that gives us almost 1.7 metres.

All this applies to the perfect – black and sharp – shadow of the helicopter, and not to the penumbra, which is weak and hazy.

Incidentally, our computation shows that if instead of a helicopter we had a small balloon about 1.7 metres in diameter, there would be no perfect shadow. All we would see would be a hazy penumbra.