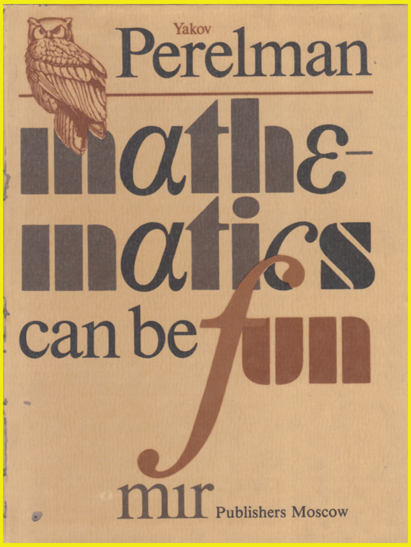

Continued....

Part by part of this book will be published in the Vidurava issues for the benefit of the readers.

In November 2014 issue we published Questions 9 - 13 of the above publication. In this issue, you can read Questions 14- 18.

14. Guessing a Number Without Asking Anything

Tell your friend to take any three-digit number not ending with a nought (but one in which the difference between the extreme digits is not less than 2) and ask him to reverse the order of the digits. After that he must subtract the smaller number from the bigger and add the result to itself, only written in reversw order. Without asking him anything, you tell him the final result.

For instance, if the number he thought of is 467, then he must do the following:

$$467;\quad 764;\quad \begin{array}{r} 764 \\ -467 \\ \hline 297 \end{array} \qquad \begin{array}{r} 297 \\ +792 \\ \hline 1089 \end{array}$$

It is this final result that you tell your friend. How do you find it? Let us examine the problem in general. Let us take a number with digits a,b, and c, with a bigger than c by at least two units. Here is how it will look:

$100a + 10b + c.$

The number with the digits reversed will look thus:

$100c + 10b + a.$

The difference between the first and the second will equal:

$99a – 99c.$

We then do the following transformations:

$$\begin{aligned} 99a - 99c &= 99\,(a-c) = 100\,(a-c) - (a-c) \\ &= 100\,(a-c) - 100+100 -10+10 - a+c \\ &= 100\,(a-c-1) +90+ (10-a+c). \end{aligned}$$

Consequently, the difference consists of the following three digits:

Number of hundreds : $a - c - 1$,
Number of tens: 9,
Number of units : $10 + c – a$.

The number with the digits reversed will look thus:
$100\,(10+c-a) + 90 + (a-c-1)$.

Adding the two expressions

$$\begin{array}{l} \quad 100\,(a-c-1) + 90 + 10+c-a \\ + \\ \quad 100\,(10+c-a) + 90 + a-c-1, \end{array}$$

We get
$100 \times 9 + 180 + 9 = 1089.$

And so, irrespective of the choice of digits a,b, and c, we always get the same number : 1089. It is not difficult, therefore, to guess the result of these calculations: you have known it all along.
Naturally, you should not pose this problem twice to the same man. He will guess the trick.

15. Who has it?

This clever trick requires three little things that can be put into a pocket: a pencil, a key, and a penknife will do very well. In addition to that, put a plate with 24 nuts (draughts or domino pieces or matches will do just as well) on the table.
Having completed these preparations, ask each of your three friends to put one of the three things into his pocket, the first the pencil, the second the key and the third the penknife. This they must do in your absence, and when you return to the room, you guess correctly where each object is.
The process of guessing is as follows: on your return (i.e. after each has concealed the object) you ask your friends to take care of some nuts, you give one nut to the first, two to the second and three to the third. Then you leave the room again, telling them that they must take more nuts – the one who has the pencil should take as many nuts as he was given the first time; the one with the key twice as many as he has been given; and the one with the penknife four times the number. The rest, you tell them, should remain in the plate.
When they have done that, they call you into the room. You walk in, look at the plate and anounce what each of your friends has in his pocket.
The trick is all the more mystifying

since you do it solo, so to speak, without any assistant who could signal to you secretly. There is really nothing tricky in the riddle – the whole thing is based on calculation. You guess who has each of the things from the number of nuts remaining in the plate. Usually there are not many of them left, from one to seven, and you can count them at one glance. How, then, do you know who has what thing? Simple. Each different distribution of the three objects leaves a different number of nuts in the plate. Here is how it is done.

Let us call your three friends Dan, Ed, and Frank, or simply D,E,F. The three things will be as follows: the pencil, a, the key, b, and the penknife, c. The three objects can be distributed among the trio in just six ways:

D	E	F
a	b	c
a	c	b
b	a	c
b	c	a
c	a	b
c	b	a

There can be no other combinations, the table above exhausts all of them.

Now let us see how many nuts remain after each combination:

DEF Number of nuts taken			Total	Remainder
abc 1+1=2;	2+4=6;	3+12=15	23	1
acb 1+1=2;	2+8=10;	3+6=9	21	3
bac 1+2=3;	2+2=4;	3+12=15	22	2
bca 1+2=3;	2+8=10;	3+3=6	19	5
cab 1+4=5;	2+2=4;	3+6=9	18	6
cba 1+4=5;	2+4=6;	3+3=6	17	7

You will see that the remainder is each time different. Knowing what it is, you will easily establish who has what in his pocket. Once again, for the third time, you leave the room, look at your notebook into which you have written the table above (frankly speaking, you need only the first and last columns). It is difficult to memorise this table, but then there is really no need for that. The table will tell where each thing is. If, for example, there are five nuts remaining in the plate, the combination is bca, i.e.

Dan has the key,
Ed has the penknife, and
Frank has the pencil.

If you want to succeed, you must remember how many nuts you gave each of your three friends (the best way is to do so in alphabetic order, as we have done here).

16. A Chain of 28 Dominoes
Can you lay out the 28 dominoes in a chain, observing all the rules of the game?

17. The Two Ends of a Chain
The chain of the 28 dominoes begins with five dots. How many dots are there on the other end of the chain?

18. A Domino Trick

Your friend takes a domino and asks you to build a chain with the remaining 27, affirming that this can be done no matter what piece is missing. After that he leaves the room.

You lay out the dominoes in a chain, and find that your friend is right. What is even more wonderful is that your friend, without seeing your chain, can tell you the number of dots on each of the end dominoes.

How can he know that? And why is he so certain that a chain can be built up of any 27 dominoes?

Answer 16 - 18

16. To simplify the problem let us set aside all the seven double dominoes: 0-0, 1-1, 2-2, etc. There will remain 21 pieces with wach number repeated six times. For instance, there will be four dots (on one half of the domino) on the following size dominoes:
4-0, 4-1, 4-2, 4-3, 4-5, and 4-6.

We thus see that each number is repeated an even number of times. It is obvious that these dominoes can be linked up. And when this is done, when the 21 pieces are arranged in an uninterrupted chain, then we insert the seven double dominoes between dominoes ending with the same number of dots, i.e. between two 0's, two 1's, two 2's, etc. After that all the 28 dominoes will be drawn into the chain according to the rules of the game.

17. It is easy to prove that a chain of 28 dominoes must end with the same number of dots as it begins. Indeed, if it were not so, the number of dots on the ends of the chain would be repeated an odd number of times (inside the chain the numbers always lie in pairs). However, we know that in a complete set each number is repeated eight times, i.e. an even number of times. Therefore, our assumption about the unequal number of dots on the ends of the chain is wrong: the number of dots must be the same (in mathematics arguments of this sort are called reduction ad absurdum).

Incidentally, this property of the chain has another extremely interesting aspect: namely, that the ends of a 28-piece chain can always be linked to form a ring. Thus, a complete domino set may be arranged according to the rules of the game both into a chain with loose ends or into a ring.

The reader may be interested to know how many ways there are of arranging this ring or chain. Without going into tiresome calculations we can say that there is a gigantic number of ways of doing this. It is exactly 7 959 229 931 520 (representing the value of 213 x 38 x 5 x 7 x 4231).

18. The solution of this problem is similar to the one described above. We know that the 28 dominoes can always be arranged to form a ring. Therefore, if we take one piece away,

(1) the remaining 27 will always form an uninterrupted chain with loose ends, and (2) the numbers on the loose ends of this chain will always be those on the two halves of the domino taken away. Concealing a domino, you can always tell beforehand the number of dots on each end of the chain.