Mathematical Modeling and its Applications in Day-to-Day life

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A couple of decades ago, Mathematics meant some sort of introduction to mathematical reasoning courses, in which we were supposed to learn how to prove mathematical theorems. During that time the answers to questions like, **"What is this good for?"**; **"After these the structures of logical arguments?"**; **"Is it a technique we can use to prove many theorems?"**; **"Is it a cool argument/result?"**; or **"Are we going to use it to prove something else?"**. Later many mathematicians would say that this is what mathematicians do, and that mathematics is about-proving theorems.

Today mathematics is at the other extreme. Our goal is to learn how to use mathematics to understand/explain/describe/predict aspects of the real world. This is what scientists and engineers do, they use mathematics as a language that is more precise, clearer, and more useful for their purposes than 'natural' language. Despite the judgement asserted at the end of the previous paragraph, this is the context in which the bulk of mathematical language is used.

There is perhaps an analogy with natural language: A small fraction is used for literary purposes, e.g., poetry, where a substantial emphasis is on language for language's sake, while by far the majority of language is more utilitarian, e.g., conversation, or newspaper writing. Just as many mathematicians would emphasize the importance of proofs, many more literature professors study and teach poetry than newspaper articles.

Applications drove the early creation of mathematics: counting, measuring land, navigation, etc., and pure mathematics continues to develop in tandem with applications: low-dimensional topology and gauge theory, algebraic geometry and string theory, graph theory and analysis of the Internet, etc.

That is what mathematicians do in modern times; they try to describe and analyze some real world phenomena mathematically, which may require learning some new mathematics, and even proving a few theorems along the way. This process is called Mathematical Modeling. Keeping this idea in mind, let us move to get experience about mathematical modeling.

What is Mathematical Modeling?

Mathematical Modeling is the process of creating a mathematical representation of some phenomenon in order to gain a better understanding of that phenomenon. It is a process that attempts to match observation with symbolic statement. During the process of building a mathematical model, the modeler will decide what factors are relevant to the problem and what factors can be de-emphasized. Once a model has been developed and used to answer questions, it should be critically examined and often modified to obtain a more accurate reflection of the observed reality of that phenomenon. In this way, mathematical modeling is an evolving process; as new insight is gained, the process begins again as additional factors are considered.

Building a mathematical model

Building a mathematical model for real world problems can be challenging, yet an interesting task. A thorough understanding of the underlying scientific concepts is necessary and a mentor with expertise in problem topic is invaluable. It is also best to work as part of a team to provide more brainstorming power. In industry and engineering, it is common practice for a team of people to work together in building a model, with the individual team members bringing different areas of expertise to the problem.

Although problems may require very different methods of solution, the following steps outline a general approach to the mathematical modeling process:

- **Identify the problem**, define the terms in your problem, and **draw diagrams** where appropriate.

- Begin with a **simple** model, **stating the assumptions** that you make as you focus on particular aspects of the phenomenon.
- **Identify important variables and constants** and determine how they relate to each other.
- **Develop the equation(s)** that express the relationships between the variables and constants.

Verifying and refining a model

Once the model has been developed and applied to the problem, our resulting model solution must be analyzed and interpreted with respect to the problem. The interpretations and conclusions should be checked for accuracy by answering the following questions:

- Is the information produced reasonable?
- Are the assumptions made while developing the model reasonable?
- Are there any factors that were not considered that could affect the outcome?
- How do the results compare with real data, if available?

In answering these questions, we may need to modify our model. **This refining process should continue until we obtain a model that agrees as closely as possible with the real world observations of the phenomenon that we have set out to model.**

Variables and parameters

Mathematical models typically contain three distinct types of quantities: *output variables, input variables, and parameters* (constants). Output variables give the model solution. The choice of what to specify as input variables and what to specify as parameters is somewhat arbitrary and often model dependent. Input variables characterize a single physical problem while parameters determine the context or setting of the physical problem. For example, in modeling the decay of a single radioactive material, the initial amount of material and the time interval allowed for decay could be input variables, while the decay constant for the material could be a parameter. The output variable for this model is the amount of material remaining after the specified time interval.

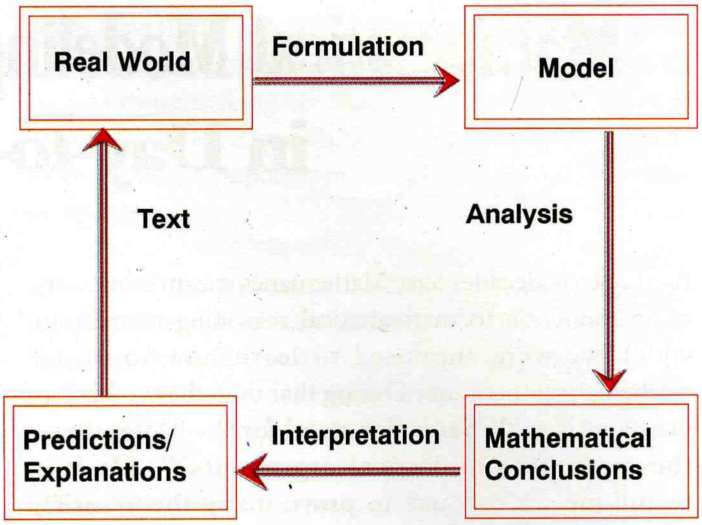

Figure 1: Process of Mathematical Modeling

With this brief idea about mathematical modeling, let us try to understand what kind of fields can be described using mathematical models. Any kind of real world problems can be described using mathematical models, but the following fields can be listed due to the importance of these in day-to-day life.

- Engineering
- Finance
- Economics
- Industry
- Biology
- Medicine
- Ecology
- Physiology
- Chemical processes
- Environmental problems
- Sports
- Wild life Management
- Management

Now we would like to outline a few real world phenomenon in which we are practising mathematics.

Tsunami prediction

How does mathematical models help us to understand the behaviour of a physical phenomenon like a **tsunami**? We can talk about equations that describing this phenomenon. This means the equations provide a relationship between variables such as wave speed or wavelength and physical parameters such as gravity and ocean depth. Such a relationship can be used to approximate the future behaviour of the wave itself.

For example, the usual approximation for wave speed in the open ocean is based on the formula, speed is proportional to the square root of the force of gravity times the ocean depth. ***Such a quantifiable relationships allows scientists to generate warnings of approaching tsunamis, with predictions of both arrival times and wave size, so that costal communities may be called for evacuations.***

Figure 2: Tsunami waves

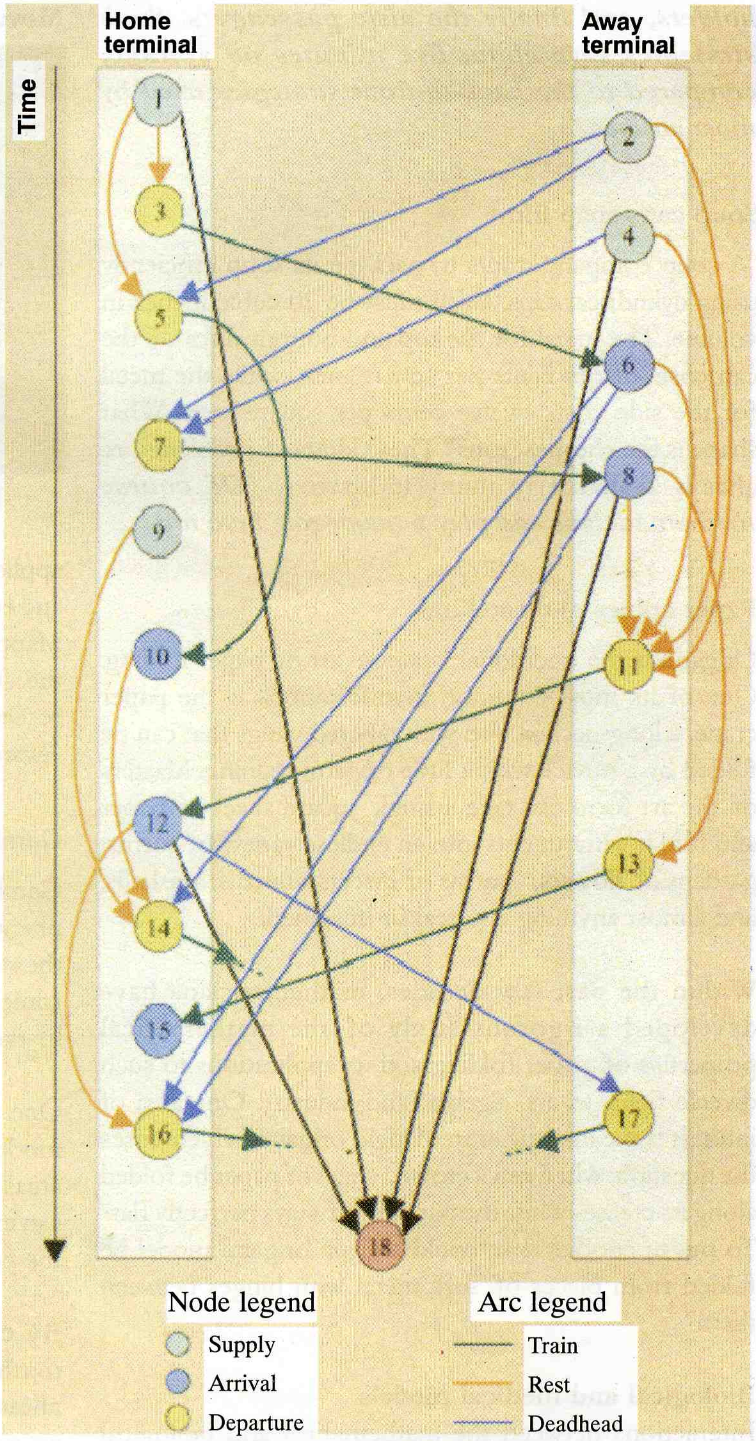

Figure 3: Network model for the scheduling problem

Airplane boarding

When boarding an aircraft we often hear the announcement - "Now passengers boarding rows 15 to 18, …now boarding rows 20 and up." We have all heard this monotonous call as we patiently wait to board a plane. When your row finally win this airplane boarding lottery, you hurriedly walk down the gangway only to wait on the aisle while your fellow passengers stow their luggage, and also allow more time for those in aisle seats to get up to allow the window and middle-seat passengers to slide into their cramped, coach-class quarters. The whole process seems to go on forever. Why cannot this process be done faster and more efficiently?

Using a mathematical model and a computer simulation, we compared a variety of boarding strategies. By putting passengers in a random order within their boarding group and running a large number of simulations (3,000 for each strategy), we determined the mean, best, and worst-case boarding times. The results were surprising, given our experiences with the supposedly efficiency-conscious airline industry.

The quickest way to get passengers onto a plane is to board them from the outside in, starting with the window passengers, then the unlucky middle-seat

holders, and finally the aisle passengers. Such strategies save about five minutes on average, compared to the back-to-front strategies used by most airlines.

Soup cans soap film

"A soup company wants to package its soup efficiently using cylindrical cans. Each must be 20 cubic inches in volume. The metal for the top and bottom discs of the can costs fifteen cents per square inch, while the metal for the side costs twelve cents per square inch. What shape is the cheapest can?" These kinds of problems are always available in many industries. ***Of course mathematicians can play a major role here too.***

Paper cranes and satellites

Origami is the traditional Japanese art of paper folding. One of its most common manifestations is the paper crane, a long-necked bird with tapered wings that can be folded by a novice with a little origami training. Masters of the art form can take a single square sheet of paper and fold it without cuts into an endless variety of forms: gardens of flowers, swarms of insects, animals, seashells, and almost anything else real or imagined.

Within the past few decades, mathematicians have developed a rigorous study of the mathematical properties of paper folding and its applications to such diverse fields as art, algebra, and industry. One area of interest is the investigation of rigid origami, which poses the question: when can a creased sheet of paper be folded along its creases while the paper itself stays perfectly flat? To put in another way, could a given origami model be folded from pieces of stiff metal with hinges between them?

Biological and medical models

Interactions between the mathematical and biological sciences have been increasing rapidly in recent years. Both traditional topics, such as population and disease modeling (especially cancer modeling), and new ones such as those in genomics arising from the accumulation of DNA sequence data, have made biomathematics an exciting field. The best predictions of numerous individuals and committees have suggested that the area will continue to be one of great growth.

Move industry

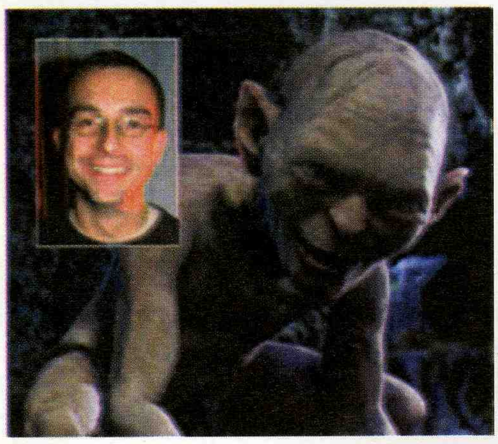

Figure 4: Lord of the Rings

Today, people use mathematical modeling techniques in the move industry too. Image modeling is a research field, which is rapidly growing and which can be applied in countless ways to the industry, medical science and entertainment. Henrik Wann Jensen invented Photon Mapping at DTU, the technique that was used to simulate the skin on Gollum in "**Lord of the Rings**". For his research Henrik Wann Jensen received an Academy Award at the Oscar ceremony.

Gambling games

Gambling games like roulette are good models for many phenomena involving chance, for example, investing in the stock market. It is easier to analyse games involving a roulette wheel than investments involving the stock market but the same ideas are involved.

One can give thousands of examples for mathematical models in various different fields. The above kinds of areas are good examples of the kinds of questions we can examine if we have a good model, and these illustrate the power of modeling.

By considering all these facts, one can think of mathematical modeling as an activity or process that allows a mathematician to be a chemist, an ecologist, an economist, a physiologist, a biologist ...etc.

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