

SHORT REVIEW

The non-linear dynamics of over-end unwinding yarn package: theory and experiment

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Abstract: Over-end unwinding has been proved to be the most optimal process to transfer yarn from one package to another in order to improve the quality and the characteristics of subsequent processes like warping and weaving. The highly nonlinear behaviour of this unwinding process results in the variation in quality of the final product and if this behaviour is not controlled, the variation becomes more pronounced. This non-linear behavior should be well understood to select the optimum process parameters for a given operation and subsequent processing. This study was done to analyze the effects of various process parameters on the tension distribution and balloon profile and to experimentally validate the mathematical model set forth for predicting the behaviour of over-end unwinding. The variables used for the study purpose are Unwinding Speed, Mass Linear Densities, Package Diameters, and Guide-eye Distances.

Keywords: Over-end unwinding, ring spinning balloon theory, rotating yarn loops, yarn balloons.

INTRODUCTION

Yarns are fundamental to the production of over 90% of consumer and industrial textiles. In order to improve the quality and characteristics, or to improve the subsequent process capability (warping, weaving, etc.), the yarn is transferred from one package to another. The most optimal process to achieve this objective is over-end unwinding. It has been observed over the years that over-end unwinding of yarn from the helically wound stationary packages is highly non-linear¹. Each yarn element during the unwinding performs a complex motion that involves movement along the yarn axis and rotation about the axis of the package.

There has always been a trend to achieve higher processing speeds without the loss of efficiency and

hence obtain higher production. This leads to the higher magnitude and wider fluctuations of the unwinding tension. In this case, once a critical speed is passed, variation in tension, occurs when uncontrolled. The problem is not only due to high level of speeds², but also due to change in package diameter, variation in yarn mass linear density, etc.

In case of high-speed unwinding, this optimization requires keeping the magnitude and pattern of variation in tension to a minimum. This non-linear behaviour of the unwinding should be well understood in order to achieve the optimum solution. In over-end unwinding, the yarn from the helically wound stationary package on the creel is unwound through a guide eye, *O*, (Figure 1). The guide eye is located along the axis of the package. This region, i.e., the yarn path from the package to the guide eye, can be divided into at least two regions. The region from the guide eye to the point where the yarn leaves the package, i.e. the lift-off point, *L*, is termed as “*Balloon region*” (or region 1). This is the portion of yarn, which flies out of the package and forms a balloon, by attaining a considerable rotational speed around the package axis. Before flying out, the yarn starts moving away from its original stationary position on the package, which is called the unwind point, *U*. This portion of the yarn, from the lift-off point, *L*, to the unwinding point, *U*, is called to be in the “*Sliding region*” (or region 2). If the tension of the yarn at the unwind point, *U*, is less than or equal to the residual tension (i.e. the tension at which the yarn is lying at rest in the package), these two regions are sufficient to analyze the problem. But if the tension at *U*, is larger than that of the residual tension, a third region, (or region 3) from the unwind point, *U*, to the stationary point, *S*, comes into play. This suggests

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that the yarn tension at stationary point, S, must then be equal to the residual tension in the yarn package³. During unwinding (usually at constant speed, v), the yarn moves back and forth on the package surface. Hence, all three points, i.e. the lift-off point, L, the unwind point, U, and the stationary point, S move periodically backward and forward on the package surface. Here, the backward (front-to-back) unwinding indicates the moving away of the unwind point, U, from the guide eye, and the forward (back-to-front) unwinding is defined by the movement of the unwind point, U, towards the guide eye.

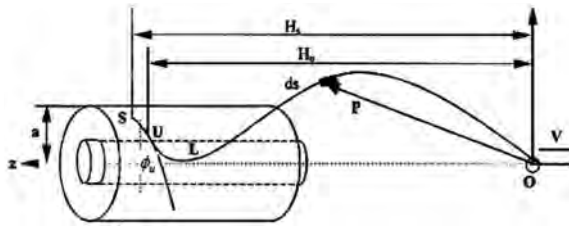


Figure 1: Illustration of over-end unwinding

Theoretical analysis of over-end unwinding process

The over-end unwinding, the yarn motion in one end fixed (at the guide-eye) and the other end rotating around the package with a certain rotational speed was analyzed. Fraser⁴ formulated the differential equations for spinning balloons by describing the yarn motion in a stationary path on the surface of a cylinder. He obtained solutions for the distribution of yarn tension and normal force on the yarn, as the yarn moves on the surface of the cylinder. The solutions were accurate for a constant distance between the yarn lift-off point and the guide-eye. They obtained the steady state and non-steady state solutions for multiple loop balloons with cylindrical packages. The limitations of their work can be enlisted as:

1. No results have been obtained when the ratio of the balloon height to the package radius < 5 .
2. The rate of change of the balloon height relative to its length has always been assumed to be 1.0 at the lift-off point.
3. The dragging of yarn on the package surface has not been considered. Kong *et al.*⁵ has investigated the steady-state motion of inextensible yarns during unwinding and slightly modified the models to include the effects of non-zero wind angle packages and numerically solved them to predict the yarn tension and the geometric properties of the unwinding balloon and the sliding yarn in the backward unwinding. A critical review of the literature reported on unwinding follows.

Balloon region

Ghosh *et al.*⁶ have shown that air resistance acting on the ballooning yarn does not increase its tension. It has been assumed that each component of the yarn in the balloon was affected by the tension and the air drag, the longitudinal component of the drag being neglected and the perpendicular component being proportional to the square of normal component of velocity. The assumptions made in their theory were:

1. The yarn was perfectly flexible, inextensible and uniform throughout in linear density.
2. The gravitational and internal elastic forces of each element of the yarn in the balloon, were negligible.
3. The longitudinal component of the air resistance is small compared to the tension and hence was neglected; however, the perpendicular component was assumed to be proportional to the square of the normal component of velocity.
4. The rate of fluctuations of balloon shape and size was sufficiently slow to allow their neglect in formulating the governing equations of motion.
5. Steady state motion of the yarn, i.e. the winding on the package is at right angles to the axis.

Mathematical formulation unwinding from cylindrical package

The governing equations developed in previous research on cylindrical packages have been modified to analyze the problems of unwinding from conical package, such as warping and weaving. As shown in Figure 1, the yarn path can be divided into three regions: the balloon region (OL, region 1) where yarn flies in the air, the drag region (LU, region 2) where yarn moves on the surface of package, and the stationary region (US, region 3) on the surface of package where yarn tension varies due to the static friction. The equation of motion of yarn in region 1 and 2 can be written as:

$$m \left\{ D^2 \underline{R} + 2\omega \underline{k} \times D \underline{R} + D\omega \underline{k} \times \underline{R} + \omega^2 \underline{k} \times (\underline{k} \times \underline{R}) \right\} = \frac{\partial}{\partial S} \left(T \frac{\partial \underline{R}}{\partial S} \right) + \underline{F} \quad (1)$$

Where, m is the mass linear density of the yarn, $\underline{R}(S,t) = x\underline{i} + y\underline{j} + z\underline{k}$ is the position vector of point P, $T(S,t)$ is the yarn tension. For region 1, \underline{F} is the air drag force per unit length of the yarn, and for region 2, \underline{F} is frictional drag force per unit length of yarn. Figure 1 shows yarn being removed from a stationary cylindrical package by over-end withdrawal through a guide eye located at a fixed point on the package axis. The yarn is assumed to be perfectly flexible, uniform and inextensible. The

unwinding balloon rotates in still air. Let O_{xyz} be a Cartesian co-ordinates system that rotates with an angular velocity $\omega \underline{k}$ about the z axis, which coincides with the package axis. The origin of the coordinates system O is at the eyelet.

For the unwinding balloon the air drag force

$$\underline{F} = -D_n |\underline{v}_n| \underline{v}_n \tag{2}$$

where D_n is the air drag coefficient. The yarn velocity vector is defined as

$$\frac{d\underline{R}}{dt} = \underline{v} = D\underline{R} + \omega \underline{k} \times \underline{R} \tag{3}$$

with normal component of \underline{v}

$$\underline{v}_n = \frac{\partial \underline{R}}{\partial S} \times \left(\underline{v} \times \frac{\partial \underline{R}}{\partial S} \right) \tag{4}$$

The differential operator D represents the total time derivative and is given by

$$D = \frac{\partial}{\partial t} + \frac{\partial}{\partial S} \frac{\partial S}{\partial t} = \frac{\partial}{\partial t} - v_0 \frac{\partial}{\partial S} \tag{5}$$

Where v_0 is the withdrawal speed. In steady state all quantities are assumed independent of time and

$$D = -v_0 \frac{\partial}{\partial S} \tag{6}$$

Equation 1 is subject to boundary condition at the eyelet and lift-off point. At the eyelet ($S=0$)

$$\underline{R}(0) = 0 \tag{7}$$

$$\text{or } x(0) = 0, y(0) = 0, z(0) = 0 \tag{8}$$

In addition, we chose the rotating reference frame so that the slope at eyelet in the xOz plane :

$$\frac{dy}{ds}(0) = 0 \tag{9}$$

At the liftoff point ($S = S_l$),

$$\sqrt{x(S_l)^2 + y(S_l)^2} = a, \quad z(S_l) = H_l \tag{10}$$

Where a is the package radius and H_l is the balloon height. If we assume that the friction force between yarn on the package is negligible, the yarn then flies into the balloon tangent to the package surface at the liftoff point.

$$\frac{d}{ds} \left(x(S_l)^2 + y(S_l)^2 \right)_{s=S_l} = 0 \tag{11}$$

The boundary conditions¹ at the guide eye ($x = y = z = 0$) and at the lift-off point, where the yarn is tangential to the package surface. For each value of yarn tension at the guide eye, balloon height, and the wind angle (at the lift-off), the two point boundary value problem was solved successfully. By reducing motion equations (1) to dimensionless forms they¹ suggested that the non-dimensional tension fluctuation at the guide eye could be obtained as (tension parameter τ)

$$\tau = \frac{T_0}{m v^2 / g} \tag{12}$$

where, T_0 is the tension at the guide eye, with non-dimensional air-resistance coefficient. The yarn unwinding dynamics presented a more general set of equations as shown below.

$$m \{ \ddot{x} - 2\omega \dot{y} - \omega^2 x \} = \frac{\partial}{\partial S} \left(T \frac{\partial x}{\partial S} \right) + D_n l v^2 \tag{13}$$

$$m \{ \ddot{y} + 2\omega \dot{x} - \omega^2 y \} = \frac{\partial}{\partial S} \left(T \frac{\partial y}{\partial S} \right) + D_n l v^2 \tag{14}$$

$$m \ddot{z} = \frac{\partial}{\partial S} \left(T \frac{\partial z}{\partial S} \right) + D_n l v^2 \tag{15}$$

where, $x = -v \frac{\partial x}{\partial S} + \frac{\partial x}{\partial t}$ and $v = v^2 \frac{\partial^2 x}{\partial S^2} - 2v \frac{\partial^2 x}{\partial S \partial t} + \frac{\partial^2 x}{\partial t^2}$

(similarly for y and z) and l is the length of yarn from eye guide to lift off point. To achieve non-dimensionality of the equations, they introduced a new parameter $k = \frac{v}{a\omega}$, which is unity in case of steady state

unwinding (as ϕ_l is zero at lift - off). However, when the winding on the original package was oblique, i.e. when ϕ_l is not zero, the time derivatives $\frac{\partial x}{\partial t}, \frac{\partial^2 x}{\partial t^2}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}$ etc. are not zero and then k is no longer strictly constant (or unity). Approximated parameter k can be expressed in terms of ϕ_l ,

$$k = \frac{1 \pm \sin \phi_l \frac{dS_l}{dz_l}}{\cos \phi_l} \tag{16}$$

where S_l , z_l were arc length of yarn and balloon height from guide eye to the lift-off point, respectively. Here positive sign indicates decrease in balloon height, and negative sign indicates an increase in balloon length. Hence the approximated⁷ solution to this non-steady state problem from a steady state problem. When ϕ_l is not too large (i.e., $\phi_l < 30^\circ$). Also, as long as ϕ_l remains constant, the second time derivative can also be neglected. If ϕ_l is positive for forward unwinding and negative for backward unwinding, the parameter k may be expressed as:

$$k = \frac{v}{a\omega} = \frac{1 - \sin \phi}{\cos \phi} \tag{17}$$

By solving the problem numerically with the boundary conditions at the guide eye and the lift-off point and applying specified yarn wind angle and balloon height, the solution of the differential equations are found as the form of a quadratic equation to relate tension at guide eye (T_o), balloon length (z_l) of single balloon (for constant p and ϕ_l), i.e.

$$T_o = mv^2 \left\{ A + B(z_l/a)^2 \right\} \tag{18}$$

where, A and B are constants dependent on the air-resistance parameter p and yarn-winding angle ϕ_l . As was evident from the relation that as the balloon height (z_l) increased, the tension at the guide eye increased as well. Similarly, variations of tension with the yarn-winding angle ϕ_l were important and cannot be ignored. For multiple balloons, which were similar in kind to single balloons suggested that the yarn tension to be approximated by

$$T_o = mv^2 \left\{ A^{(1)} n^{-\frac{1}{2}} + B^{(1)} (z_l/na)^2 \right\} \tag{19}$$

where, $A^{(1)}$, $B^{(1)}$ represent the coefficients appropriate to a single balloon of length (z_l/n). Researchers have suggested that for a given distance from package to the guide eye, and a given withdrawal speed there would be an upper limit to the number of balloons and a corresponding lower limit to the tension. Modified model⁸, with the inclusion of gravitational force and tangential air drag to the yarn in the equations of motion that were ignored and numerical solution of the equations for the tension distribution in the balloon, and a relation

$$T = T_l + \frac{m\omega^2}{2} (R^2 - r^2) \tag{20}$$

They assumed T_l (yarn tension at the lift-off point) to be known. Here, R and r are the radius of the package and balloon respectively, m is the mass linear density and

ω = angular velocity of the balloon. They suggested that the tension at any given point of the ballooning portion of the yarn depends on two components. The first is the static tension and the second is dynamic component (depending on the angular velocity, the yarn count and the distance between the point on the balloon and the axis of rotation). According to the relation, the maximum tension occurs at the point where the radius is equal to

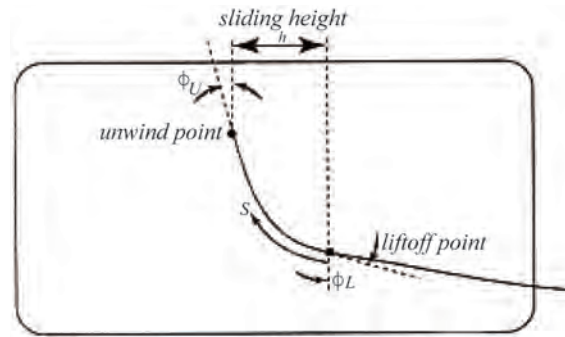


Figure 2: Illustration of drag Region

zero. In this case, the guide eye represents that point and in the case of multiple balloons, there will be more than one point to represent this condition. He suggested that if T_l is known, then the shape of the balloon, its angular velocity, and T can be predicted.

Sliding region

As defined earlier, during unwinding, before flying out (or lift-off), the yarn starts moving away from its original stationary position on the package, which is called the unwind point, U (Figure 2). This portion of the yarn, from the lift-off point, L, to the unwinding point, U, is called to be in the “Sliding region” (or region 2). This sliding of the yarn on the package surface determines, to some extent, the balloon shape and the tension distribution in the yarn, which has been studied by many researchers². The yarn length as well as the shape of the curve in this region on the package surface varies (Figure 2). This variation causes increase or decrease in the frictional drag force between the yarn and the package depending on the length of the yarn. The motion of equations over the surface of the cylinder for the ‘sliding region’ (region 2) as:

$$\left. \begin{aligned} 0 &= \frac{\partial T}{\partial \sigma} - \mu N \sin \frac{\phi}{2} \\ mv^2 \frac{\partial \phi}{\partial \sigma} &= T \frac{\partial \phi}{\partial \sigma} - \mu N \cos \frac{\phi}{2} \\ mv^2 (1 - \cos \phi)^2 / a &= -N + \frac{T}{a} \cos^2 \frac{\phi}{2} / a \end{aligned} \right\} \tag{21}$$

where, N is normal reaction force per unit length of

yarn, a is radius of the cylinder, T is yarn tension, μ is the coefficient of friction between yarn and package, σ is arc length of the yarn measured from the unwind point (transition point) towards the guide eye and ϕ is the angle of wind of the stationary coils on the package (with $\phi = 0$ at the unwind point, and steady state). Lofer³ studied the unwinding process with the conditions in the zone of yarn separation from the package and derived the relation for initial tension at the separation point (or unwind point) T_0 as:

$$T_0 = m v_e^2 + \frac{Q}{2 \sin \phi_0 / 2} \quad (22)$$

where, m is the mass linear density of the yarn, v_e is the constant withdrawal speed, Q is the friction force between the yarn and the underlying layer on the package and ϕ_0 is the unwind angle. He modified (22) and gave the new equation to calculate the tension as:

$$T_0 = m v_e^2 + \frac{Q}{2 \sin \phi_0 / 2} \frac{\cos^2 \phi_0 / 2}{\cos^2 \phi / 2} \quad (23)$$

where, T is the yarn tension and ϕ is the yarn angle at an arbitrary point from the lift-off point to the unwind point. He⁴ concluded that the yarn tension increases along the sliding section when the yarn slides, but does not rise above double the initial tension (i.e. T_0). He also claimed that, given the unwind point tension, the yarn tension on the package can be determined by the yarn angle on the package without any error.

Integrated analysis

The unwinding process involves the yarn to move from unwind point to the lift-off point and then fly into the balloon until the guide eye. These two regions (Balloon region and Sliding region) from lift-off point to the guide eye and from unwind point to the lift-off point, are distinguished by the nature of forces applied to them during unwinding. Kong *et al.*⁵ have given the equations of motion (in the vector form) that were applicable for both the regions.

$$m \left\{ D^2 \underline{R} + 2\omega \underline{k} \times D \underline{R} + \omega^2 \underline{k} \times (\underline{k} \times \underline{R}) \right\} = \frac{\partial}{\partial S} \left(T \frac{\partial \underline{R}}{\partial S} \right) + \underline{F} \quad (24)$$

where, D is total time derivative differential operator given by, $D = \frac{\partial}{\partial t} - v \frac{\partial}{\partial S} \{ \underline{R}(S, t) = r e_r + z \underline{k} \}$ is the position vector of a yarn segment at time t relative to an origin O at the guide eye, s is the yarn arc length measured from the guide eye measured along the yarn, v is the constant withdrawal speed, m is mass linear density, ω is the angular velocity of the rotating frame and T is the tension in the yarn. The vector \underline{F} is the drag force per unit length acting on the yarn, which is due to air drag in the balloon

region or frictional drag on the surface of the package. Since D is the total time derivative, $D \underline{R}$ is the velocity of the yarn segment and $D^2 \underline{R}$ is its acceleration relative to the rotating frame. The second term in the braces on the left side of the (24) is the Coriolis acceleration and the third is the centripetal acceleration. They have given a set of boundary conditions for the guide eye and unwind point, and continuity conditions at the lift-off point to integrate the two regions and get solution from the guide eye to the unwind point. Unwinding has been defined to be a periodic motion with period Γ . The derived dimensionless equations in which every term with time derivative multiplied by a parameter ϵ has been given by

$$\epsilon = \frac{a}{v \Gamma} = (a / 2H) \sin \phi_u$$

Kong *et al.*⁵ have given the equation of motion in a first approximation as:

$$m \left\{ \frac{\partial^2 \underline{R}}{\partial S^2} - 2 \underline{k} \times \frac{\partial \underline{R}}{\partial S} + \underline{k} \times (\underline{k} \times \underline{R}) \right\} = \frac{\partial}{\partial S} \left(T \frac{\partial \underline{R}}{\partial S} \right) + \underline{F}$$

by using shooting method, if the boundary conditions at guide eye and lift-off point are given. Different solutions for single and multiple balloons have been found. It has also been found that as the number of balloons increased, the tension at the guide eye decreased. It varied periodically with the tension at the guide eye, T_0 , and over the length of the package. It varied with the package radius from full to empty package. But then it cannot be equal to the residual tension T_{res} of the yarn lying at rest on the package, as T_{res} is not expected to vary. Different boundary conditions have been used at the unwind point as the wind angle ϕ_u , as a boundary condition by specifying $\left. \frac{dz}{ds} \right|_{s=s_u} = \sin \phi$ at the unwind point with

approximation as long as $\sin \phi_u \approx \phi_u$ was valid. The guide eye tension T_0 does not change significantly with frictional coefficient, based on experimental results. However, during unwinding of a layer of a given radius, the unwinding tension T_0 at the guide eye has been determined by the specified tension at the unwind point T_u , assuming steady state unwinding, and defined $\lambda = \frac{\omega a}{v}$ (where λ is relative rotational parameter).

Doing so has enabled the comparison of theoretical and experimental results and suggestion that the value of λ is >1 for backward unwinding, and <1 for forward unwinding. Multiple solutions for different balloon shapes have also been found. The value of λ , as suggested by Kong *et al.*⁵ could be determined experimentally by

measuring the angular velocity of the rotating yarn, and then using it in the theoretical model for the comparison with the experimental results.

Stationary region

Goswami¹ has assumed that the static frictional force at the stationary point (F_s) was zero and that it attained the maximum value at the unwind point (F_m) (between them it is distributed linearly). The equation in the third region (stationary region) has been given by Ghosh *et al.*⁶ as

$$\frac{d}{dS} \left(T \frac{dR}{dS} \right) + F_s \frac{dR}{dS} + N e_r = 0$$

where, N is the normal force per unit length of the yarn and T is the tension in the yarn. They have described the three-region problem and that resulted in a three point BVP. Multiple solutions with different balloon shapes have also been found and Ghosh *et al.*⁶ have concluded:

1. Yarn tension at the guide eye is nearly independent on wind angle and is not significantly affected by the yarn-package drag co-efficient. It is directly proportional to the distance between the stationary point on the package surface and the guide eye.
2. Residual tension is a critical factor affecting balloon shapes and tension distribution, and in determining the transition between the two-and three-region problem (when $T_u = T_{res}$, three-region problem reduced to two-region).
3. In two-region analysis, when residual tension is higher than a certain limiting value, the yarn admits a kink at the unwind point, whereas in three-region analysis, the wind angle is considered constant. Ghosh *et al.*⁶ have suggested solutions using wind angle and the residual tension, but have used the height of stationary point as a boundary condition, which cannot be known a priori.

CONCLUSION

The effects of parameters such as yarn linear density, unwinding speed, package diameter, and guide eye distance on the unwinding tension and balloon profile were studied and the results were verified with the theoretical model presented. It was observed that the unwinding balloons have different shapes depending on the processing conditions, which sometimes cannot be calculated or predicted from theoretical models. From the images captured, it was observed that the number of

balloons increases as the yarn unwinding speed increases as well as with the increase in distance between the front of the package and the guide eye. Yarn tension increases with the increase in the unwinding speed (with the same guide eye distance), also giving rise to increased number of balloons. This indicates that the angular velocity, centrifugal force and air resistance increase with the unwinding speed. In agreement with the theory, highest possible tensions were observed at the single balloon formations. The direction of unwinding affects the tension at the guide eye tension increasing as the unwind point moves away from the guide eye, i.e. with the backward unwinding, being highest in the far end zone. Nevertheless, it affects the balloon profile, i.e. number of balloons increases as the diameter decreases, which was predicted by the theory. The unwinding tension increases as the guide eye distance increases, therefore a movable guide-eye system could be designed. The distance between the package front and the guide eye should be selected in such a way that it avoids formation of single balloons at higher speeds. Since the rotational speed of the yarn plays a significant role in determining the tension distribution, the package could be placed on a rotating package holder. This would reduce the rotational speed of the yarn and hence the unwinding tension to a certain limit.

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