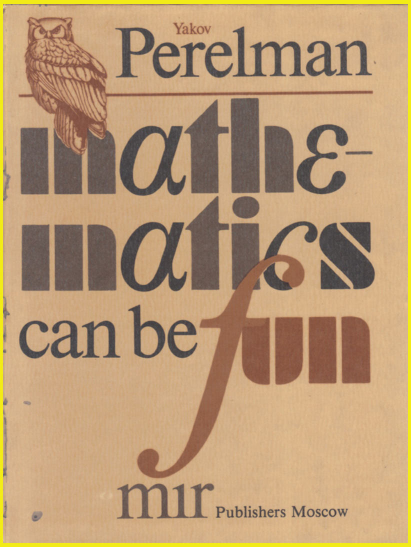


Continued....

Part by part of this book will be published in the Vidurava issues for the benefit of the readers.

In June 2015 issue we published Questions 14 - 18 of the above publication. In this issue, you can read Questions 19- 23.

19. A Frame

Figure 4 shows a square frame made up of dominoes in accordance with the rules of the game. The sides are equal in length, but not in the total number of dots. The top and left sides add up to 44 points each, while the other two to 59 and 32 points, respectively.
Can you build a square frame in which each side will have 44 points?

20. Seven Squares

It is possible to build a four-domino square in such a way as to have the same number of dots on each side, as shown in Fig. 5: there are 11 dots on each side.

Fig.4

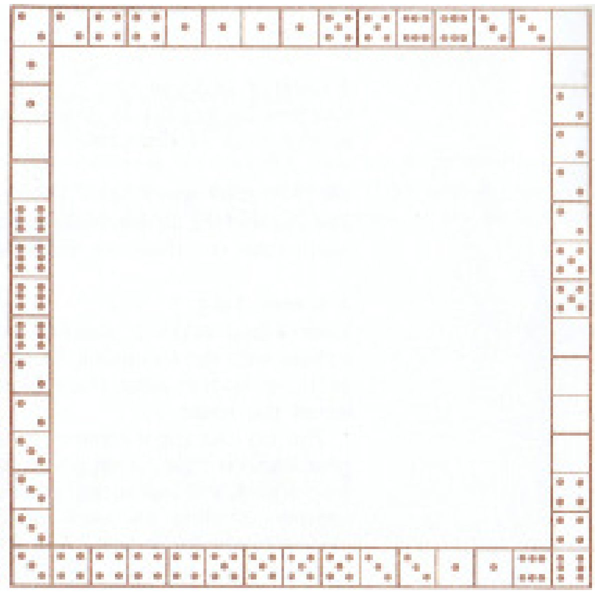


Fig.5

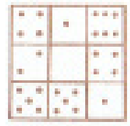
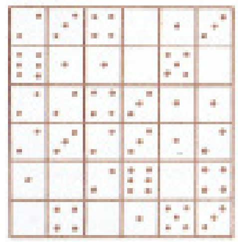


Fig.6



Can you make seven such squares out of the 28 dominoes? It is not necessary for all the sides of the seven squares to have an identical number of dots, just the four sides of each square.

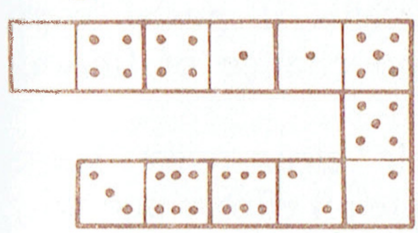
21. Magic Squares

Figure 6 depicts a square of 18 dominoes. The wonder of it is that there are 13 dots in every one of its rows, columns, and diagonals. From time immemorial these squares have been called “magic”.
Arrange several other 18-piece magic squares, but with a different number of dots. Thirteen is the lowest total in a row and 23 the highest.

22. Progression in Dominoes

Figure 7 shows six dominoes

Fig.7



arranged according to the rules of the game, with the number of dots increasing by one on each successive piece: four on the first, five on the second, six on the third, seven on the fourth, eight on the fifth and nine on the sixth.
A series of numbers increasing (or decreasing) by the same amount is known as “arithmetic progression”. In our case, each number is greater than the preceding by one, but there can be any other “difference”. The task is to arrange several other six-piece progressions.

The Fifteen Puzzle
The story of the well-known square shallow box with 15 blocks numbered 1 to 15 inclusive is an extremely interesting one, though very few players know it. Here is what W.Ahrens, German mathematician and draughts expert, wrote about it:

“In the late 1870s there appeared in the United States a new game.’The Fifteen Puzzle’. Its popularity spread fast and wide, and it soon became a real social calamity.
“The craze hit Europe too. One came across people trying to solve

the puzzle everywhere, even on public transport. Office workers and shop salesman became so absorbed in working it out that their employers had to forbid the game during working hours. Enterprising people took advantage of the mania to arrange large-scale tournaments."

The puzzle made its way even into the German Reichstag. The well-known geohrapher and mathematician Siegmund Gunther, a Reichstag deputy at the time of the craze, recalled seeing his grey-haired colleagues bending thoughtfully over the little square boxes.

"In Paris the game was played in the open air, on the boulevards, and soon spread from the capital to the provinces.'There was no rural homestead where this spider had not woven its web', was how one French author described the craze.

"The fever was apparently at its highst in 1880, but mathematicians soon defeated the tyrant by proving that only half of the numerous problems it posed were solvable. There was absolutely no chance of finding a solution for the rest.

"The mathematicians made it clear why some problems remained unsolved despite all efforts and why the organizers of tournaments were not afraid of offering huge prizes for their solution. In this the inventor of the puzzle, Sam Loyd, surpassed everyone. He asked a New York newspaper owner to offer $1000 to anyone who would solve a certain variant of the Fifteen Puzzle, and when the publisher hesitated, Loyd said he would pay the sum himself.

Lyd was well known for his clever conundrums and brain-teasers. Curiously enough, he could not get a U.S. patent for his puzzle. According to reguations, a person applying for one was required to submit a 'working model'. At the patent Offic he was asked if the puzzle was solvable, and Loyd had to admit that mathematically it was not.'In that case', the official said,'there can be no working model and without it there can be no patent.Loyd left it at that, but there is no doubt that he would have been much more insistent could he have but foreseen the unusual success of his invention."

Here are some facts about the puzzle, as told by the inventor himself:

"Puzzle enthusiasts may well remember how, in the 1870s, I caused the world to rack its brain over a box with moving blocks, which became known as 'The Fifteen Puzzle'. Thirteen of the blocks were arranged in regular order (Fig.8) and only two, 14 and 15, were not (Fig.9). The task was to shift one block at a time until blocks 14 and 15 were brought into regular order.

"No one won the $1000 prize offered for the first correct solution, although people worked tirelessly at it.

There are many humorous stories told of tradesmen who were so absorbed in the puzzle they forgot to open their shops, and of respected officials who spent nights seeking for a way to solve the problem. People just would not give up their search for the solution, being confident of success. Navigators ran their ships against reefs, locomotive engineers missed stations, and farmers chucked up their ploughs."

We shall acquaint the reader with the rudiments of the puzzle.

On the whole, it is extremely complicated and closely connected with one of the sections of advanced algebra ("the theory of determinants"). Here is what Ahrens wrote about it:

"The job is to shift the blocks by using the blank space so as to arrange all 15 in a regular order, i.e. to have block 1 in the upper left-hand corner, block 2 to the right of it, block 3 next to block 2, and block 4 in the upper right-hand corner; to have blocks 5,6,7, and 8 in this order in the next row, etc. (see Fig.8).

"Imagine for a moment that the blocks are all placed at random. It is always possible to bring block 1 to its
correct position through a series of moves.

"It is equally possible to shift block 2 to the next
square without touching block 1. Then, without touch-
mg blocks 1 and 2, one can move blocks 3 and 4 into their
places. If, by chance, they are not in the last two vertical rows, it is easy to bring them there and to achieve the desired result. The top row 1, 2, 3, and 4 is now in order and in our subsequent operations we shall leave these four blocks alone. In the same way we shall try to put in order the row 5, 6, 7, and 8; this is also possible. Further, in the next two rows it is necessary to bring blocks 9 and 13 to their correct positions. Once in order, blocks 1, 2, 3, 4, 5, 6, 7, 8, 9, and 13 are not shifted any more. There remain six squares one of them blank and the other five occupied by blocks 10, ii, 12, 14, and 15 in pelimell order. It is always

Fig. 8

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

possible to shift blocks 10, ii, and 12 until they are arranged correctly. When .this is done, there will remain blocks 14 and 15 in proper or improper order in the bottom row (Fig. 9). In this way, as the reader can verify himself, we come to the following result:

"Any original combination of blocks can be brought into the order shown in Fig. 8 (position I) or Fig. 9 (position II).

"If a combination, which we shall call C for short, can be rearranged into position I, then it is obvious that we can do the reverse too, i. e. rearrange position I into combination C. After all, every move can be reversed: if, for instance, we can shift block 12 into blank space, we can bring it back to its former position just as well.

"Thus, we have two series of combinations: in the first we can bring the blocks into regular order (position I) and in the second into position II. And conversely, from regular order we can obtain any combination of the first series and from position II any combination of the second series. Finally, any one of the two combinations of the same series may be reversed.

Fig. 9

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

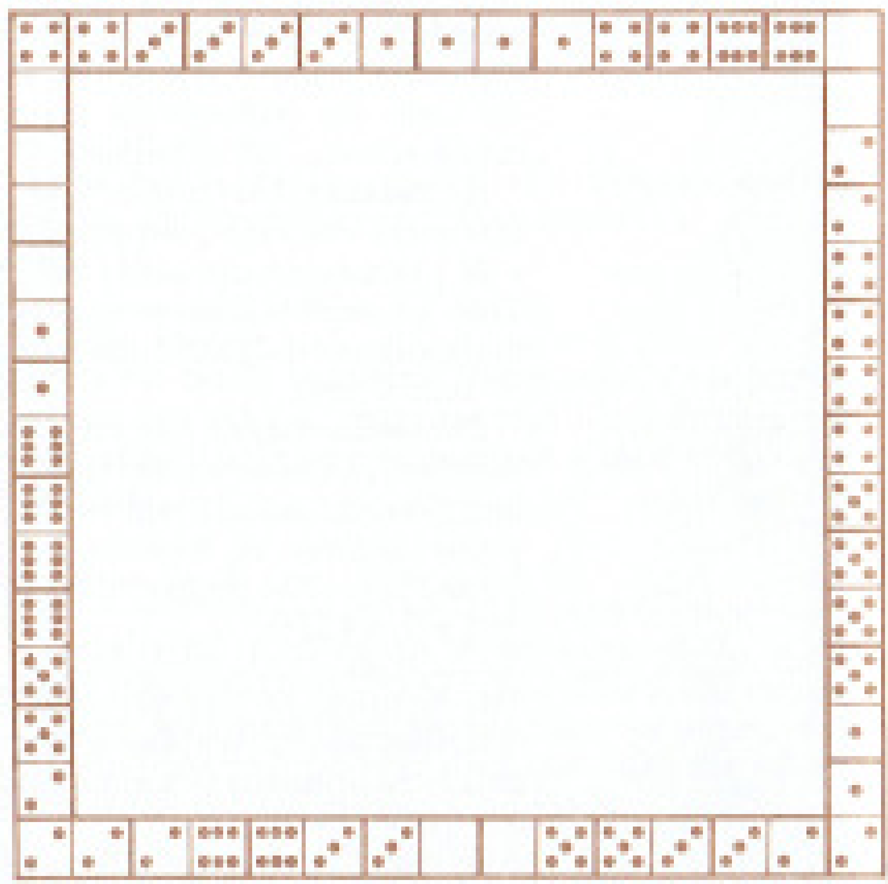
Fig. 10

	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Fig. 11

4	8	12	
3	7	11	15
2	6	10	14
1	5	9	13

Fig. 12



"Is it possible to transform position I into position II? It may definitely be proved (without going into detail) that no number of moves is capable of doing that. Therefore, the huge number of combinations of blocks may be classed into two series, the first series where the blocks can be arranged in regular order, i. e. the solvable; and the second series where in no circumstances can the blocks be brought into regular order, i. e. the insoluble, and it is for the solution of these positions that huge prizes were offered.

"Is there any way of telling to what series the position belongs? There is, and here is an example.

"Let us analyse the following position. The first row of blocks is in order and the second, too, with the exception of block 9. This block occupies the space that rightfully belongs to block 8. Therefore, block 9 precedes block 8.

Such violation of regular order is termed 'disorder'. Our analysis further shows that block 14 is three spaces ahead of its regular position, i. e. it precedes blocks 12, 13, and 11. Here we have three 'disorders' (14 before 12, 14 before 13, and 14 before 11). Altogether we have 1 + + 3 = 4 'disorders'. Further, block 12 precedes block 11, just as block 13 precedes block 11. That gives us another two 'disorders' and brings their

Fig. 13

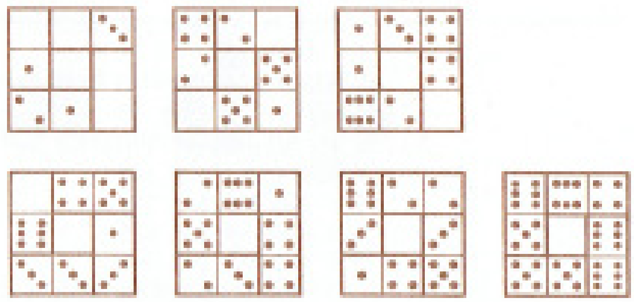
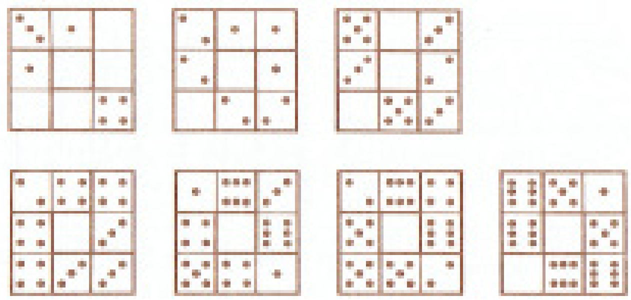


Fig. 14



total to 6. In this way we determine the number of 'disorders' in each position, taking good care to vacate beforehand the lower right-hand corner. If the total number of 'disorders', as in this case, is even, then the blocks can be arranged in regular order and the problem is solvable. If, on the other hand, the number of 'disorders' is odd, then the position belongs to the second category, i. e. it is insoluble (zero disorder is taken as even number).

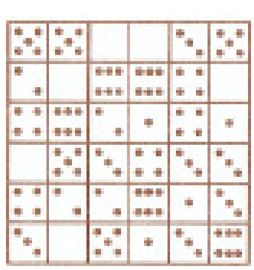
"Mathematical explanation of this puzzle has dealt a death blow to the craze. Mathematics has created an exhaustive theory of the game, leaving no place whatever for doubts. The solution of the puzzle does not depend on guesswork or quick wit, as in other games, but solely on mathematical factos tht predetermine the result with absolute certainty."

Let us now turn to some problems in this field. Below are three of the solvable problems, made up by the inventor.

23 .The First Problem

In Figure 9 arrange the blocks in regular order, leaving the upper left-hand corner blank (as in Fig. 10).

Fig. 15



Answer 19 - 23

19. The total number of dots on the four sides of the unknown square must equal 44 x 4 = 176. i.e. 8 more than the total number of dots on all the dominos (168). This is because the numbers at the vertices of the squares are counted twice. This determines that the total of the numbers at the vertices is 8 and that helps to find the necessary arrangement (although its discovery nevertheless remains quite troublesome). The solution is shown in Fig. 12.

20. Here are two of the many solutions of this problem. In the first (Fig. 13) we have:
1 square with a total of 3, 2 squares with a total of 9,
1 square with a total of 6, 1 squares with a total of 10,
1 square with a total of 8, 1 squares with a total of 16,
In the secong (Fig. 14) we have:
2 square with a total of 4, 2 squares with a total of 10,
1 square with a total of 8, 2 squares with a total of 12,

21. Figure 15 s a specimen of a magic square with a total of 18 dots in each row.

22. Here, as an example, are two progressions with a difference of 2:
(a) 0-0, 0-2, 0-4, 0-6, 4-4 (or 3-5), 5-5 (or 4-6),
(b) 0-1, 0-3 (or 1-2), 0-5 (or 2-3), 1-6 (or 3-4)! 3-6 (or 4-5), 5-6.
There are altogether 23 six-domino progressions. The starting dominoes are:
(a) For progressions with a difference of 1:
0-0 1-1 2-1 2-2 3-2
0-1 2-0 3-0 3-1 2-4
1-0 0-3 0-4 1-4 3-5
0-2 1-2 1-3 2-3 3-4
(b) For progressions with a difference of 2: 0-0, 0-2, 0-1.

23. This problem may be solved by the following 44 moves:
14, 11, 12, 8, 7, 6, 10, 12, 8, 7, 4, 3, 6, 4, 7, 14, 11, 15, 13, 9, 12, 8, 4, 10, 8, 4, 14, 11, 15, 13, 9, 12, 4, 8, 5, 4, 8, 9, 13, 14, 10, 6, 2, 1.